LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SEVENTH SERIES.]

MARCH 1929.

XLIX. On the Electromagnetic Field of an Electron.—The Electron as a Gravitational Phenomenon. By D. Meksyn*.

§ 1. The Properties of an Electron and the Special Theory of Relativity.

A CCORDING to the present Electrodynamics, an Electron is an atom of charge in its own Electromagnetic Field; the latter could be represented as a system of stresses self-balanced in the whole field and resolved into

a force acting on the Electron itself.

We make an attempt to discard this picture of an Electron, and we assume that an Electron represents the same entity as a neutral mass, with the only difference that, whereas matter, or energy, is located in a particle in a very small region, in an Electron it is spread all over the space according to the law

 $\frac{A}{r^4}$, or, what turns out to be the same, that an Electron is a Field of Gravitation whose potential is not $\frac{m}{r}$, but $\frac{m}{r^2}$; there are no stresses in the Field of an Electron †.

It is true that it is now accepted in some quarters that the stresses have no physical reality, and are only convenient mathematical conceptions. It makes, however, no difference

* Communicated by H. T. Flint, D.Sc.

 $[\]uparrow$ The Electron is assumed to have a special localization about a point from which r is measured, and the law of extension applies outside this central localization.

whether we consider them to be real or not, if only we accept the consequences which follow from this conception, as, for instance, the existence of an Electromagnetic Force, the connexion between the mass and Energy etc.

The "abolition" of stresses clears up from the outset some of the difficulties connected with the theory of Electrons.

- 1. If there are no stresses (or the stresses are mathematically equal to zero), an Electron does not possess an Electric Field, and hence it is unnecessary to "explain" the existence of an Electron. An Electron is a substance which is in equilibrium just as a mass particle.
- 2. From the dynamics of the Theory of Relativity it follows that mass and Energy are connected by

$$m = \frac{W^0}{c^2}. \qquad (1)$$

This is, however, not borne out by the Electron Theory, because, according to the latter, the momentum of an Electron is equal * to

$$G = \frac{4 \,\mathrm{W}^{0} u}{3c \,\sqrt{c^{2} - u^{2}}}; \qquad (2)$$

hence the mass is equal to

If, however, we discard the stresses, the momentum will be

$$G = \frac{uW^0}{c\sqrt{c^2 - u^2}} \cdot \dots \cdot (4)$$

and the mass

$$m = \frac{W^0}{c^2}, \dots, \dots, \dots, \dots$$
 (5)

or in complete agreement with dynamics.

3. The Law of Energy for an Electron is not satisfied for an accelerated Motion.

If we assume that the mass of an Electron is wholly electromagnetic, its momentum will be

$$G_x = \frac{4W^0u}{3c^2\sqrt{1 - \frac{u^2}{c^2}}}, \dots (6)$$

^{*} M. v. Laue, 'Die Relativitätstheorie,' 1921, p. 134.

and Energy

$$W = \frac{c^2 + \frac{1}{3}u^2}{c^2 \sqrt{1 - \frac{u^2}{c^2}}} W^0. \qquad (7)$$

The Equation of Energy is

$$u\frac{dG_x}{dt} - \frac{dW}{dt} = \frac{d}{dt} \frac{\sqrt{1 - \frac{u^2}{c^2}}}{3} W^0; \quad . \quad . \quad . \quad (8)$$

or in order to preserve the Law of Energy, it is necessary to admit the existence of some additional mechanical Energy of value *:

$$\frac{\sqrt{1-\frac{u^2}{c^2}}}{3}W^0. \qquad (9)$$

In our case this difficulty also disappears because the Momentum and Energy are equal to

G =
$$\frac{uW^0}{c^2\sqrt{1-\frac{u^2}{c^2}}}$$
, $W = \frac{W^0}{\sqrt{1-\frac{u^2}{c^2}}}$, . (10)

and hence the Law of Energy is satisfied.

§ 2. The Gravitational Field of an Electron.

1. Classical Theory.—According to the classical theory, the Gravitational Potential of continuous matter is given by

$$\nabla^2 \phi = -4\pi k \rho, \qquad (11)$$

where ρ is the density of matter and k the constant of Gravitation.

In our case

$$\rho = \frac{e^2}{8\pi c^2 r^4}; \qquad (12)$$

hence

$$\nabla^2 \phi = -\frac{ke^2}{2c^2r^4}$$

and

$$\phi = -\frac{ke^2}{4c^2r^2}. \qquad (13)$$

* M. v. Laue, *ibid.* p. 226. 2 F 2 2. The Theory of Relativity.—The Gravitational potential could be directly obtained from Einstein's solution for Gravitational waves *.

We give here a somewhat different solution for a particular

case in which

$$ds^2 = -(1+2\Omega)(dx^2 + dy^2 + dz^2) + (1-2\Omega) dt^2, \quad (14)$$

where Ω does not depend upon time.

The Equations of Gravitation are

$$G_{\mu\nu} = -8\pi k (T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T), \quad . \quad . \quad (15)$$

where T_{uv} is the Energy tensor and

$$\mathbf{T} = g^{\mu\nu} \mathbf{T}_{\mu\nu}. \qquad (16)$$

Now, neglecting the terms of the order Ω^2 , we have

$$G_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\frac{\partial^2 g_{\mu\nu}}{\partial x_\rho \partial x_\sigma} + \frac{\partial^2 g_{\rho\sigma}}{\partial x_\mu \partial x_\nu} - \frac{\partial^2 g_{\mu\sigma}}{\partial x_\rho \partial x_\nu} - \frac{\partial^2 g_{\rho\nu}}{\partial x_\mu \partial x_\sigma} \right), (17)$$

which for our case become †

$$G_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \frac{\partial^2 g_{\mu\nu}}{\partial x_{\sigma} \partial x_{\rho}} = -\frac{1}{2} \nabla^2 g_{\mu\nu} = \nabla^2 \Omega, \quad . \quad . \quad (18)$$

and the Law of Gravitation is

$$\nabla^2 \Omega = -8\pi k (T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) ; \quad . \quad . \quad (19)$$

in our case all $T_{\mu\nu} = 0$ except

$$T_{44} = g_{44} T_4^4 = \frac{e^2}{8\pi r^4 c^2} \dots (20)$$

and

$$T = g_{\mu\nu}T^{\mu\nu} = T_4^4 = \frac{e^2}{8\pi r^4 c^2}.$$
 (21)

Now the right side of (19) is equal for $\mu = \nu = 1, 2, 3$

$$-8\pi k \left(+ \frac{1}{2} \frac{e^2}{8\pi r^4 c^2} \right) = -\frac{ke^2}{2r^4 c^2}, \quad . \quad . \quad (22)$$

and for $\mu = \nu = 4$

$$-8\pi k \left(\frac{e^2}{8\pi r^4 c^2} - \frac{1}{2} \frac{e^2}{8\pi r^4 c^2}\right) = -\frac{1}{2} \frac{ke^2}{r^4 c^2}, \quad . \quad (23)$$

or the same as (22).

* A. Einstein, 'Über Gravitationswellen,' Berlin, Sitzungsberichte, 1918, p. 154.

† A. S. Eddington, 'The Mathematical Theory of Relativity,' 1924,

p. 102.

Our assumption as to the value of $T_{\mu\nu}$ is consistent with Einstein's Law of Gravitation.

The Equation of Gravitation becomes

$$\nabla^2\Omega = -\,\frac{1}{2}\,\frac{ke^2}{r^4c^2}$$

and

$$\Omega = -\frac{ke^2}{4r^2c^2}, \qquad (24)$$

or the same as in the classical theory.

If we compare this solution with Nordström's and Jeffery's *, we notice an important discrepancy: our solution is only half of Jeffery's, which is equal to

$$\Omega = -\,\frac{1}{2}\,\frac{e^2}{r^2}$$

(if allowance is made for the factor $\frac{1}{4\pi}$ in the expression of

the Electromagnetic Energy).

This is due to the value of the Energy Tensor $T_{\mu\nu}$, which was taken by Jeffery under the assumption of existence of stresses. We neglect here the ordinary gravitational contribution to $r:\frac{m}{r}$ which arises from the central localization.

§ 3. Relative and Invariant Mass.

If we accept Maxwell's stresses, a well-known difficulty arises in the explanation of the mass of an Electron †.

We have to discriminate between the relative mass T44

and the invariant mass $g_{\mu\nu}T^{\mu\nu}$.

Now it appears that the latter, for Maxwell's stresses, is equal to zero, and hence some additional assumption becomes necessary in order to explain the equality of relative and invariant mass for an Electron at rest.

If we discard the stresses, the invariant mass $g_{\mu\nu} T^{\mu\nu}$ becomes equal for an Electron at rest to its relative mass.

§ 4. Two Electrons in Space and the Electromagnetic Force.

We have seen that an Electron does not possess an electromagnetic field; the latter appears only if we have two or more electrons in space, and is wholly due to the increase (or decrease) of energy of space above (or below) the sum of energies of the two Electrons.

^{*} A. S. Eddington, *ibid.* p. 185. † *Id. ibid.* p. 183.

To define the laws of the electromagnetic force we must find the energy of two Electrons.

It is clear that, as both energies are spread over the space,

some interaction may arise between them.

We know that the gravitational field of two material points and its energy are found (to the first approximation) by superposition of the two fields and their energies. Hence, if we translate the energy of an Electron as an equivalent energy of a gravitational field of matter, we can make some inferences about the laws of an electromagnetic field by applying the same superposition.

This interpretation can be carried out as follows:—

The energy density of an Electron is equal to

where
$$\mathbf{E} = \frac{e^2}{8\pi r^4} = \frac{1}{8\pi} \left[\left(\frac{\partial \mathbf{V}}{\partial x} \right)^2 + \left(\frac{\partial \mathbf{V}}{\partial y} \right)^2 + \left(\frac{\partial \mathbf{V}}{\partial z} \right)^2 \right],$$

$$\mathbf{V} = \frac{e}{r}.$$
(25)

By Green's transformation we have

$$\iiint \nabla \nabla^2 \nabla \, dx \, dy \, dz + \iiint \left\{ \left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2 \right\}$$

$$\times \, dx \, dy \, dz + \iiint \nabla \frac{dV}{dn} \, dS = 0. \quad (26)$$

The last term vanishes at the boundary, and as $V = \frac{e}{r}$, the first term is equal to

$$\iiint \nabla^2 \nabla \, dx \, dy \, dz = -4\pi \iiint \nabla \rho \, dx \, dy \, dz = -4\pi e \nabla, \quad (27)$$
whence
$$E = \frac{1}{2} e \nabla. \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Hence we come to the conclusion that the energy of an Electron is equal to the energy of a mass ϵ in a gravitational field V, or, from (25), the energy at every point of the field is proportional to the square of its equivalent gravitational Force.

The latter is what we usually call the Electrostatic force of an Electron.

Now, if we have two Electrons to which we ascribe electric forces

$$X_1, Y_1, Z_1, X_2, Y_2, Z_2,$$

the resultant force will be, according to the theory of gravitation,

 $X_1 + X_2$, $Y_1 + Y_2$, $Z_1 + Z_2$,

and their common energy

$$\frac{1}{8\pi} \iiint [(X_1 + X_2)^2 + (Y_1 + Y_2)^2 + (Z_1 + Z_2)^2] dx dy dz
= \frac{1}{8\pi} \iiint (X_1^2 + Y_1^2 + Z_1^2) dx dy dz
+ \frac{1}{4\pi} \iiint (X_1 X_2 + Y_1 Y_2 + Z_1 Z_2) dx dy dz
+ \frac{1}{8\pi} \iiint (X_2^2 + Y_2^2 + Z_2^2) dx dy dz (29)$$

In (29) the first and the last term represent the energies of the two Electrons, and the middle term the energy of their interaction.

Of this energy

$$W = \frac{1}{4\pi} \iiint (X_1 X_2 + Y_1 Y_2 + Z_1 Z_2) \, dx \, dy \, dz \quad . \quad (30)$$

is due the so-called ponderomotive force of the field. If W=0, there is no mechanical action between two such Electrons.

The derivation of (29) cannot be considered as rigorous. We take the expression of W as one which has to be confirmed a posteriori rather than found by deduction.

For the general case of two electromagnetic fields we assume that the extra energy is, as in the case of two electrostatic fields, equal to the scalar product of the six vectors of force, i. e.

$$W = \iiint [(X_1 X_2 + Y_1 Y_2 + Z_1 Z_2) - (\alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2)] \times dy \, dx \, dz \quad . \quad (31)$$

where XYZ, $\alpha\beta\gamma$ are the electric and magnetic forces.

§5. An Electron in an Electrostatic Field.

The extra energy is

$$W = \frac{1}{4\pi} \iiint \left(\frac{\partial V_1}{\partial x} \frac{\partial V_2}{\partial x} + \frac{\partial V_1}{\partial y} \frac{\partial V_2}{\partial y} + \frac{\partial V_1}{\partial z} \frac{\partial V_2}{\partial z} \right) dx dy dz,$$

or integrating by parts and omitting the surface integral,

$$W = -\frac{1}{4\pi} \iiint V_2 \nabla^2 V_1 dx dy dz = e_1 V_2.$$

§ 6. An Electron in a Magnetic Field.

The extra energy W (31) vanishes identically (because every term is equal to 0); hence there is no mechanical interaction between an electrostatic and magnetostatic field.

§ 7. An Electron in Motion in an Electromagnetic Field (two Electromagnetic Fields).

The energy integral is equal to

$$\begin{split} \mathbf{W} = \frac{1}{4\pi} \iiint \big[\, (\mathbf{X}_1 \mathbf{X}_2 + \mathbf{Y}_1 \mathbf{Y}_2 + \mathbf{Z}_1 \mathbf{Z}_2) - (\alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2) \, \big] \\ \times dx \ dy \ dz. \end{split}$$

We express the Electromagnetic force by means of a vector and scalar potential, and obtain:

$$W = \frac{1}{4\pi} \iiint \left\{ \left[\left(-\frac{\partial \psi_1}{\partial x} - \frac{\partial F_1}{\partial t} \right) - \frac{\partial \psi_2}{\partial x} - \frac{\partial F_2}{\partial t} \right) \right.$$

$$+ \left(-\frac{\partial \psi_1}{\partial y} - \frac{\partial G_1}{\partial t} \right) \left(-\frac{\partial \psi_2}{\partial y} - \frac{\partial G_2}{\partial t} \right)$$

$$+ \left(-\frac{\partial \psi_1}{\partial z} - \frac{\partial H_1}{\partial t} \right) \left(-\frac{\partial \psi_2}{\partial z} - \frac{\partial H_2}{\partial t} \right) \right]$$

$$- \left[\left(\frac{\partial H_1}{\partial y} - \frac{\partial G_1}{\partial z} \right) \left(\frac{\partial H_2}{\partial y} - \frac{\partial G_2}{\partial z} \right) \right.$$

$$+ \left(\frac{\partial F_1}{\partial z} - \frac{\partial H_1}{\partial x} \right) \left(\frac{\partial F_2}{\partial z} - \frac{\partial H_2}{\partial x} \right)$$

$$+ \left(\frac{\partial G_1}{\partial x} - \frac{\partial F_1}{\partial y} \right) \left(\frac{\partial G_2}{\partial x} - \frac{\partial F_2}{\partial y} \right) \right] \left\{ dx \, dy \, dz \right. \qquad (32)$$

We integrate (32) by parts, and obtain a volume integral:

$$\begin{split} \mathbf{W}_{1} &= -\frac{1}{4\pi} \iiint \Bigl\{ \psi_{2} \Bigl[\frac{\partial^{2}\psi_{1}}{\partial x^{2}} + \frac{\partial^{2}\psi_{1}}{\partial y^{2}} + \frac{\partial^{2}G_{1}}{\partial t\partial y} + \frac{\partial^{2}\psi_{1}}{\partial t^{2}} + \frac{\partial^{2}H_{1}}{\partial t\partial z} + \frac{\partial^{2}F_{1}}{\partial t\partial z} \Bigr] \\ &+ F_{2} \Bigl[\frac{\partial^{2}\psi_{1}}{\partial x\partial t} + \frac{\partial^{2}F_{1}}{\partial t^{2}} - \frac{\partial^{2}F_{1}}{\partial z^{2}} + \frac{\partial^{2}H_{1}}{\partial x\partial z} + \frac{\partial^{2}G_{1}}{\partial x\partial y} - \frac{\partial^{2}F_{1}}{\partial y^{2}} \Bigr] \\ &+ G_{2} \Bigl[\frac{\partial^{2}\psi_{1}}{\partial y\partial t} + \frac{\partial^{2}G_{1}}{\partial t^{2}} + \frac{\partial^{2}H_{1}}{\partial y\partial z} - \frac{\partial^{2}G_{1}}{\partial z^{2}} - \frac{\partial^{2}G_{1}}{\partial x^{2}} + \frac{\partial^{2}F_{1}}{\partial y\partial x} \Bigr] \\ &+ H_{2} \Bigl[\frac{\partial^{2}\psi_{1}}{\partial z\partial t} + \frac{\partial^{2}H_{1}}{\partial t_{2}} - \frac{\partial^{2}H_{1}}{\partial y^{2}} + \frac{\partial^{2}G_{1}}{\partial z\partial y} + \frac{\partial^{2}F_{1}}{\partial z\partial x} - \frac{\partial^{2}H_{1}}{\partial x^{2}} \Bigr] \Bigr\} \\ &\times dx \, dy \, dz. \end{split}$$

Making use of the equation :

$$\frac{\partial \mathbf{F}_1}{\partial x} + \frac{\partial \mathbf{G}_1}{\partial y} + \frac{\partial \mathbf{H}_1}{\partial z} + \frac{\partial \psi_1}{\partial t} = 0,$$

we obtain

$$\begin{split} \mathbf{W}_1 &= -\frac{1}{4\pi} \iiint \Bigl\{ \Psi_2 \Bigl(\nabla^2 \psi_1 - \frac{\partial^2 \psi_1}{\partial t_2} \Bigr) + \mathbf{F}_2 \Bigl(\frac{\partial^2 \mathbf{F}_1}{\partial t^2} - \nabla^2 \mathbf{F}_1 \Bigr) \\ &+ \mathbf{G}_2 \Bigl(\frac{\partial^2 \mathbf{G}_1}{\partial t^2} - \nabla^2 \mathbf{G}_1 \Bigr) + \mathbf{H}_2 \Bigl(\frac{\partial^2 \mathbf{H}_1}{\partial t_2} - \nabla^2 \mathbf{H}_1 \Bigr) \Bigr\} dx \, dy \, dz. \end{split}$$

From the electron theory we have

$$\nabla^2 \psi_1 - \frac{\partial^2 \psi_1}{\partial t^2} = -4\pi \rho_1,$$

$$\nabla^2 \mathbf{F}_1 - \frac{\partial^2 \mathbf{F}_1}{\partial t^2} = -4\pi \rho_1 \frac{u}{c};$$

hence

$$W_{1} = \iiint \rho_{1} \left(\psi_{2} - \frac{u F_{2}}{c} - \frac{v G_{2}}{c} - \frac{w H_{2}}{c} \right) dx \, dy \, dz$$

$$= e_{1} \left(\psi_{2} - \frac{u F_{2}}{c} - \frac{v G_{2}}{c} - \frac{w H_{2}}{c} \right). \qquad (33)$$

This is the expression used for the potential energy in Lagrange's equations, and is obtained from Lorentz's electromagnetic force.

The remaining parts of W are

$$\begin{split} \mathbf{W}_2 &= -\frac{1}{4\pi} \iiint \left[\frac{\partial}{\partial x} \left(\psi_2 \mathbf{X}_1 + \mathbf{G}_2 \gamma_1 - \mathbf{H}_2 \boldsymbol{\beta}_2 \right) \right. \\ &+ \frac{\partial}{\partial y} \left(\psi_2 \mathbf{Y}_1 - \mathbf{F}_2 \gamma_1 + \mathbf{H}_2 \boldsymbol{\alpha}_1 \right) \\ &+ \frac{\partial}{\partial z} \left(\psi_2 \mathbf{Z}_1 + \mathbf{F}_2 \boldsymbol{\beta}_1 - \mathbf{G}_2 \boldsymbol{\alpha}_1 \right) \right] dx \, dy \, dz \\ &- \frac{1}{4\pi} \iiint \frac{\partial}{\partial t} \left(\mathbf{F}_2 \mathbf{X}_1 + \mathbf{G}_2 \mathbf{Y}_1 + \mathbf{H}_2 \mathbf{Z}_1 \right) \, dx \, dy \, dz. \end{split}$$

The first integral is a surface one, and vanishes if the expressions in brackets are of the order of $\frac{1}{r^3}$.

The last integral is a volume one, and it represents some additional energy not accounted for by Lorentz's Force.

Wheatstone Laboratory, King's College, London. L. Dr. A. N. Whitehead's Theory of Absolute Acceleration. By William Band, B.Sc., George Holt Physics Laboratory, Liverpool University *.

Foreword.

This communication criticizes the fundamental assumption involved in Dr. A. N. Whitehead's Theory of Relativity, as contrasted with Einstein's. It also points out a previously unnoticed practical consequence of the theory that is in violent contradiction with fact, quite apart from gravitational and electrical problems.

1. Introductory.—The Problem Reviewed.

CCORDING to the Special Theory of Relativity, space and time are united in a single space-time continuum, and any particular division of this continuum into space and time is not merely a property of the continuum, but of the relation of the continuum as a whole to the observer who makes the division. There is no such thing as the history of events; there is a history for every observer, the same only when the observers are in relative rest, but different when they are in relative uniform rectilinear motion. Einstein was able in his general theory to give mathematical expression to the natural generalization that all observers are equally competent to erect a coordinate system even when in relative acceleration, no observer being singled out as unaccelerated in any absolute sense. The differences between the coordinate systems erected by any two observers will depend entirely on their relative motion, and everything we could know about the latter we could deduce from the former.

Whitehead's theory is a natural reaction to this generalized theory. He accepts the verdict of the Special Relativity, that when two observers are in relative uniform rectilinear motion they will erect different coordinate systems even when momentarily at the same position (1). But he rejects, in effect, the generalization that observers in relative acceleration, but momentarily at relative rest, will also construct differing coordinate systems. According to Dr. Whitehead, an accelerated observer A will pass continually through the

^{*} Communicated by Prof. J. Rice, M.A.

various uniform velocities, and his coordinate system will be identical at any instant with that of an unaccelerated observer momentarily at rest relative to A at the instant in question (2). The observer A is accelerated in an absolute sense (3).

For brevity we shall adopt the usual convention, and consider as one group the coordinate systems used by all the observers who are only in uniform rectilinear relative velocity, and call this group the consentient set of coordinate systems.

Whitehead's theory states that there is only one consentient set, and that even accelerated observers use it (2). Einstein, on the other hand, supposes that there are an infinity of different sets, none of which can be singled out as corresponding in any absolute sense to unaccelerated observers.

2. Philosophical Considerations.

It would appear that, as Professor C. D. Broad has pointed out (4), Whitehead's theory suffers from the same drawbacks as did Newton's. The absolute basis for the measurement of acceleration will in practice be as difficult to find as was Newton's postulated basis for the measurement of uniform motion. Were our knowledge confined to one stellar universe, the Galaxy would serve well enough as a standard for rotation; but knowing as we do of other stellar universes with rotations relative to our own, we have no right to single out any one as being in any sense absolutely non-rotating. In fact, Whitehead and Newton both require that there should be a single agglomeration, finite in extent, of matter, surrounded by an infinite spread of empty space, both from the above and other (mathematical) considerations (5).

Whitehead's theory has, however, been improved in this respect by Temple's generalization, showing that the isotropic manifold can be adopted (6). The world can then be considered finite, and the total number of stellar universes also finite, so that an average could conceivably be made and adopted as a standard for rotation. While this averaging seems highly artificial, it would appear to be sufficient to raise Whitehead's theory on to a more satisfactory level than Newton's, and it seems to be logically complete, and more or

less philosophically sound.

But this last statement is considerably modified by the following considerations.

Whitehead gave the following philosophical justification

for Minkowski's unification of space and time (7).

If the instantaneous view of nature contains all the significant factors thereof, motion is merely the visible effect of a

sequence of minute changes of position; the coordinates used by an observer in motion would therefore be at any moment identical with those used by an observer at rest at the same point. But if it requires a finite time to exhibit the essential factors of nature, i.e. if we can in practice be aware of only finite events, and never limit our sense experience to instantaneous views of nature (8), we shall always be able to tell that two bodies have a relative motion even while they appear to be in the same position. The motion of a point is therefore a real factor of nature quite apart from position, and just as essential as position in determining its relations with the world as a whole. We may therefore expect quite reasonably that a relative velocity between two observers will be accompanied by a disparity between the coordinate systems used by them; for the coordinates depend on the observer's relation with the world in general.

But is it logical after this to suppose that a relative acceleration cannot cause a further disparity? If the finite time exhibits velocity as an essential factor apart from position, it will also exhibit acceleration apart from velocity, and the higher derivatives as well. But in this case we are forced to the standpoint of general relativity, that the coordinate system contains all the details pertaining to the motion of the origin, and that no origin can be singled out

as unaccelerated.

It thus appears that, when driven to its logical conclusion, the philosophy employed by Whitehead to justify the special relativity will justify the general also, and so nullify his own theory of absolute acceleration.

3. Practical Consequences.

Suppose, for the sake of further argument, that we can in fact identify the one consentient set postulated by Whitehead. Let us examine the practical consequences of supposing that the coordinates are independent of the field in which the observer is situated, and let the origin be, moreover, accelerated.

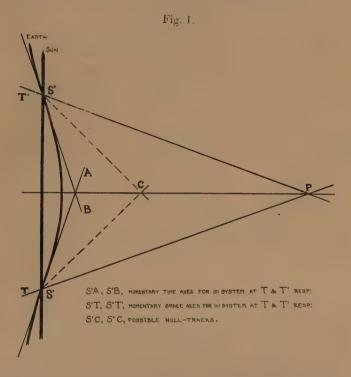
A planet is revolving absolutely about the Sun; there is an observer S at rest on the planet, who constructs coordinates (s), correct for a non-rotating planet. Imagine another observer A, permanently at rest in that member of the consentient set in which the planet, neglecting rotation, is momentarily at rest at any given instant, say T. Let A construct the coordinate system (a), one of the consentient set. Then the assumption we are examining evidently means

that at the instant T, the system (s) is momentarily identical

in all respects with the system (a).

Imagine, now, another observer B at rest in the other member of the consentient set in which the planet is momentarily at rest, say, half a year later at the instant T'. Then at T' the system (s) is in every respect identical with the system (b) of the observer B.

But since the observers A and B are moving in opposite directions, the momentary spread of space at T for A will



intersect that at T' for B (9), the region of intersection being an instantaneous plane perpendicular (10) to the line bisecting the arc of the planet's orbit that has been traced between the instants T and T', and in a direction remote from the Sun. The distance of this plane will, of course, depend upon the particular planet chosen.

Fig. 1 is simplified by including only one space dimension. It is constructed from Minkowski's principles, and shows the intersection of moments or space spreads that are

separated by half the year at the origin. We wish to investigate the intersection of moments that are separated at the origin by a vanishingly short time.

Let the planet traverse a short arc PQ which we may treat as circular. The velocity of the planet, relative to the

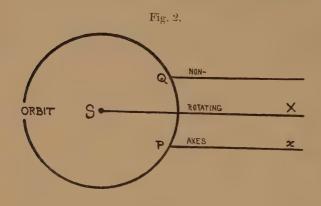
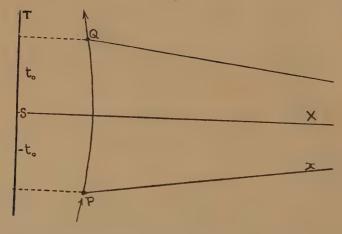


Fig. 3.



Sun, will have been reversed in a direction parallel to that radius of the orbit which bisects the arc PQ; the radius SX in fig. 2.

Fig. 2 is a timeless section in the plane of the orbit; fig. 3 is a projection on to a plane containing time and one space dimension. ST is the track of the Sun.

For the purposes of analysis take ST as the t-axis, and SX as the x-axis; the unit of "t" 3.10^{-10} seconds, and of "x" 1 cm.

The equation of the projection PQ in 3 will be

$$x = a \cos nt$$
,

where a is the radius of the orbit, and

$$n = 2\pi/[\text{period of orbit}] = 2\pi/\Gamma$$
, say.

This gives

$$dx/dt = -an\sin nt.$$

Let Px be the space axis of the planetary observer at P; its gradient in the diagram will be

$$dt/dx = an \sin nt_0,$$

and it passes through the point $(a\cos nt_0, -t_0)$. Its equation will therefore be

$$(t+t_0)/(x-a\cos nt_0)=an\sin nt_0.$$

Its intersection with the axis SX will then be given by putting t zero, i.e.

$$x = a\cos nt_0 + t_0/(an\sin nt_0),$$

which is independent of the sign of t_0 .

The space axes, or moments, for the observer at P and Q therefore respectively intersect at a distance from the Sun given by this last equation. The distance from the planet will thus be

$$x - a \cos nt_0 = \left[\frac{1}{an^2}\right] \left[nt_0/\sin nt_0\right].$$

In the limit when P and Q coincide, or t_0 approaches zero, this reduces to

$$1/an^2 = T^2/4\pi^2a$$

in a direction remote from the Sun.

Remembering the units used, we work this out as somewhat less than 10^{20} cm. in this particular case, when the

planet chosen is the Earth.

We can interpret this result only by supposing that there is some kind of vagrant horizon always on the side of the planet remote from the Sun. Successive moments close up towards this, and spread out in opposite directions. We should expect that during a total eclipse the stars revealed near the Sun's disk would exhibit a spectral shift to the blue,

the shift increasing with the distance of the star; the stars in the opposite ecliptic region would show a spectral shift to the red. The horizon is at the distance of 1020 cm.

Eddington (10) gives the distance of the further star clusters as about 10²¹ cm., while the distance of the first magnitude star Arcturus is known to be 1.6.1020 cm. Temple's generalization will merely superpose a general shift to the red, and

bring the apparent horizon nearer.

If the horizon is at 10²⁰ cm., we require a recessional motion of about 1.3.108 cm. per sec. at a distance of 4.3 lightvears—that of a Centauri, the nearest star to our solar system; this corresponds to a spectral shift relative to terrestrial sources, roughly equal to 40 times the separation between the two Na lines!

4. Conclusion.

The assumption that an accelerated observer uses at any moment a member of the consentient set of coordinate systems leads to a curious inconsistency with fact. Had Whitehead been thoroughly reactionary, and been content with Newtonian space independent of an absolute time, the moments for oppositely-moving observers would not have intersected, and the inconsistency would not have arisen. The fault lies in the attempt to compromise between the idea of absolute acceleration and that of relativity of velocity. for which, as we saw in § 2, the logical justification was doubtful.

By a reversion to a Ptolemaic system of astronomy, Whitehead might be able to explain the above results, but in that case his mathematical ingenuity would be taxed with the problem of discovering a dynamic invariant that would account for the revolution of the Sun about the Earth-not a particularly hopeful proposition.

References.

(1) Whitehead, 'The Principle of Relativity,' p. 88, "Conclusion."
(2) Loc. cit. p. 76.
(3) Loc. cit. p. 72; 'The Physical Field,' p. 88, "Rotation."
(4) 'Mind,' April 1923; Review of Whitehead, l. c., in "Critical Notices.

(5) Einstein.

(6) Proc. Phys. Soc. London, April 1924.
(7) Whitehead, 'Principles of Natural Knowledge' (?).

(8) On this point see also C. D. Broad, 'Scientific Thought,' pp. 346 f. (9) Whitehead, l. c. § 47.1.

(10) Eddington, 'The Mathematical Theory of Relativity,' § 69, § 102.

LI. The Aberration Effect on Straight-line Reproduction. By Arnold Buxton, M.A. Oxon.*.

THE theory of the reproduction of luminous points and lines in a symmetrical ontical system depends upon lines in a symmetrical optical system depends upon diffraction and the aberration residuals in the system †.

Hamilton has shown, in his 'Theory of Systems of Rays,' that the time V taken by the light to travel from the point P in the object space to Q in the image space can be expressed as a function of the coordinates of P and Q with respect to

certain axes not necessarily the same.

Clerk Maxwell, in his 'Collected Papers,' vol. ii., takes the axis of symmetry of the optical system as one axis (in our case the axis of x), and takes P in a plane through the object-origin perpendicular to the axis of symmetry, and similarly for Q in the image space. He then shows that V may now be regarded as a function of four coordinates Y, Z, η , ζ , since the other two are zero, and then from the symmetry of the optical system about the axis of x the number of variables may be reduced to three, $Y^2 + Z^2$, $\eta^2 + \zeta^2$, $Y\eta + Z\zeta$.

Further, if the first and last media of the system have refractive indices unity, then it is a property of V, the

characteristic function of Hamilton that

$$\frac{\partial V}{\partial \eta} = M_1, \quad \frac{\partial V}{\partial \zeta} = N_1, \quad \frac{\partial V}{\partial Y} = M, \quad \frac{\partial V}{\partial Z} = N,$$

where L, M, N; L₁, M₁, N₁ are the direction cosines of the incident and emergent rays respectively, i. e.

$$V = F(Y^2 + Z^2, \zeta^2 + \eta^2, Y\eta + Z\zeta).$$

Expanding V and rejecting third and higher powers of the new variables, we have

$$\begin{split} \mathbf{V} &= a_0 + a_1 (\mathbf{Y}^2 + \mathbf{Z}^2) + a_2 (\boldsymbol{\zeta}^2 + \boldsymbol{\eta}^2) + a_3 (\mathbf{Y}\boldsymbol{\eta} + \mathbf{Z}\boldsymbol{\zeta}) + a_4 (\mathbf{Y}^2 + \mathbf{Z}^2)^2 \\ &+ a_5 (\boldsymbol{\zeta}^2 + \boldsymbol{\eta}^2)^2 + a_6 (\mathbf{Y}\boldsymbol{\eta} + \mathbf{Z}\boldsymbol{\zeta})^2 + a_7 (\mathbf{Y}^2 + \mathbf{Z}^2) (\boldsymbol{\zeta}^2 + \boldsymbol{\eta}^2) \\ &+ a_8 (\mathbf{Y}^2 + \mathbf{Z}^2) (\mathbf{Y}\boldsymbol{\eta} + \mathbf{Z}\boldsymbol{\zeta}) + a_9 (\boldsymbol{\zeta}^2 + \boldsymbol{\eta}^2) (\mathbf{Y}\boldsymbol{\eta} + \mathbf{Z}\boldsymbol{\zeta}) \ \ddagger, \end{split}$$

† Clerk Maxwell, 'Collected Papers,' ii. p. 442 (18).

^{*} Communicated by the Author. † A. Buxton, Monthly Notices R. A. S. lxxxi. No. 8, p. 547 (June 1921); lxxxiii. No. 8, p. 475 (June 1923); lxxxv. No. 1, p. 78 (Nov. 1924).

with

$$N_{1} = \frac{\partial V}{\partial \zeta} = 2a_{2}\zeta + a_{3}Z + 2a_{5}\zeta(\zeta^{2} + \eta^{2}) + 2a_{6}Z(Y\eta + Z\zeta) + 2a_{7}\zeta(Y^{2} + Z^{2}) + a_{8}Z(Y^{2} + Z^{2}) + 2a_{9}\zeta(Y\eta + Z\zeta) + a_{9}Z(\zeta^{2} + \eta^{2}), \quad . \qquad (2)$$

and

$$L_1 = \sqrt{1 - M_1^2 - N_1^2}.$$
 (3)

Let X', Y', Z' be the coordinates of the point where the emergent ray meets the image plane, then

$$Y' = \eta + \frac{M_1}{I_{11}}X',$$

$$Z' = \zeta + \frac{N_1}{I_{12}}X'$$

 $Z' = \zeta + \frac{-1}{L_1}X$ from the straight-line equation

$$\frac{x-\xi}{1} = \frac{y-\eta}{M} = \frac{z-\xi}{N}$$

for the emerging ray. We must now substitute for $\frac{M_1}{L_1}$ and $\frac{N_1}{L_1}$ from (1), (2), and (3)

$$\begin{split} \frac{M_{1}}{L_{1}} &= \frac{M_{1}}{\sqrt{1 - M_{1}^{2} - N_{1}^{2}}} = M_{1} \{ 1 + \frac{1}{2} (M_{1}^{2} + N_{1}^{2}) \} \\ &= M_{1} + \frac{1}{9} M_{1}^{3} + \frac{1}{9} M_{1} N_{1}^{2} \end{split}$$

and

$$\frac{N_1}{L_1} = N_1 + \frac{1}{2}M_1^2N_1 + \frac{1}{2}N_1^3,$$

1. e.

$$\begin{split} \frac{\mathbf{M}_{1}}{\mathbf{L}_{1}} &= 2a_{2}\eta + a_{3}\mathbf{Y} + 2\eta a_{5}(\zeta^{2} + \eta^{2}) + 2a_{6}\mathbf{Y}(\mathbf{Y}\eta + \mathbf{Z}\zeta) \\ &+ 2a_{7}\eta(\mathbf{Y}^{2} + \mathbf{Z}^{2}) + a_{8}\mathbf{Y}(\mathbf{Y}^{2} + \mathbf{Z}^{2}) + 2a_{9}\eta(\mathbf{Y}\eta + \mathbf{Z}\zeta) \\ &+ a_{9}\mathbf{Y}(\zeta^{2} + \eta^{2}) \\ &+ \frac{1}{2}[a_{3}^{3}\mathbf{Y}^{3} + 6a_{3}^{2}\mathbf{Y}^{2}a_{2}\eta + 12a_{3}a_{2}^{2}\mathbf{Y}\eta^{2} + 8a_{2}^{3}\eta^{3} + 8a_{2}^{3}\eta\zeta^{2} \\ &+ 8a_{2}^{2}a_{3}\zeta\eta\mathbf{Z} + 2a_{2}a_{3}^{2}\eta\mathbf{Z}^{2} + 4a_{2}^{2}a_{3}\mathbf{Y}\zeta^{2} + 4a_{2}a_{3}^{2}\mathbf{Y}\mathbf{Z}\zeta \\ &+ a_{3}^{3}\mathbf{Y}\mathbf{Z}^{2}] \end{split}$$

$$= 2a_2\eta + a_3Y + \eta(\zeta^2 + \eta^2)(2a_5 + 4a_2^3) + 2Y^2\eta(a_6 + a_7 + \frac{3}{2}a_2a_3^2) + Y\eta^2(3a_9 + 6a_3a_2^2) + Y\xi^2(a_9 + 2a_3a_2^2) + Y^3\left(\frac{a_3^3}{2} + a_8\right),$$

introducing the simplification obtained by making OP the axis of Y or Z=0.

Similarly,

$$\begin{split} \frac{\mathbf{N}_{1}}{\mathbf{L}_{1}} &= 2a_{2}\zeta + 2a_{5}\zeta(\zeta^{2} + \eta^{2}) + 2a_{7}\zeta\mathbf{Y}^{2} + 2a_{9}\mathbf{Y}\zeta\eta \\ &+ \frac{1}{2} \left[8a_{2}^{3}\zeta^{3} + 8a_{2}^{3}\zeta\eta^{2} + 2a_{2}a_{3}^{2}\mathbf{Y}^{2}\zeta + 8a_{2}^{2}a_{3}\zeta\eta\mathbf{Y} \right] \\ &= 2a_{2}\zeta + \zeta(\zeta^{2} + \eta^{2})\left(2a_{5} + 4a_{2}^{3} \right) + \left(2a_{7} + a_{2}a_{3}^{2} \right)\mathbf{Y}^{2}\zeta \\ &+ \left(2a_{9} + 4a_{2}^{2}a_{3} \right)\mathbf{Y}\zeta\eta. \end{split}$$

Hence

$$\begin{split} \mathbf{Y}' &= \eta + \mathbf{X}' \left[2a_2\eta + a_3\mathbf{Y} + \eta(\zeta^2 + \eta^2) \left(2a_5 + 4a_2^3 \right) \right. \\ &+ 2\mathbf{Y}^2\eta(a_6 + a_7 + \frac{3}{2}a_2a_3^2) + \mathbf{Y}\eta^2(3a_9 + 6a_8a_2^2) \\ &+ \mathbf{Y}\zeta^2(a_9 + 2a_3a_2^2) + \mathbf{Y}^3\left(\frac{a_3^3}{2} + a_8\right) \right], \\ \mathbf{Z}' &= \zeta + \mathbf{X}' \left[2a_2\zeta + \zeta(\zeta^2 + \eta^2) \left(2a_5 + 4a_2^3 \right) + \left(2a_7 + a_2a_3^2 \right) \mathbf{Y}^2\zeta \right. \\ &+ \left. \left(2a_9 + 4a_2^2a_3 \right) \mathbf{Y}\zeta\eta \right], \\ \text{or} \\ \mathbf{Y}' &= \eta \left(1 + 2a_2\mathbf{X}' \right) + a_3\mathbf{X}'\mathbf{Y} + \dots \end{split}$$
 and
$$\mathbf{Z}' &= \zeta (1 + 2a_2\mathbf{X}') + \dots \end{split}$$

From these results it is evident that, if $1+2a_nX'=0$, neglecting third-order terms, Y' will be proportional to Y and Z' will be zero. Hence $X'=-\frac{1}{2a_{\parallel}}$ gives the position of the first-order image plane, and the third-order aberrations are given by the equations:

$$\delta \mathbf{Y}' = -\frac{1}{2a_{2}} \left[\eta(\zeta^{2} + \eta^{2})(2a_{5} + 4a_{2}^{5}) + 2\mathbf{Y}^{2}\eta(a_{6} + a_{7} + \frac{3}{2}a_{2}a_{3}^{2}) \right. \\ \left. + \mathbf{Y}\eta^{2}(3a_{9} + 6a_{3}a_{2}^{2}) + \mathbf{Y}\zeta^{2}(a_{9} + 2a_{3}a_{2}^{2}) \right. \\ \left. + \mathbf{Y}^{3} \left(\frac{a_{3}^{3}}{2} + a_{8} \right) \right], \quad \ldots \quad \ldots \quad (4)$$

$$\delta \mathbf{Z}' = -\frac{1}{2a_{2}} \left[\zeta(\zeta^{2} + \eta^{2})(2a_{5} + 4a_{2}^{3}) + (2a_{7} + a_{2}a_{3}^{2}) \mathbf{Y}^{2}\zeta \right. \\ \left. + (2a_{9} + 4a_{2}^{2}a_{3})\mathbf{Y}\zeta\eta \right], \quad \ldots \quad (5)$$

II. The Separate Effects of the Aberrations on the Luminous Line Integrals *.

(a) Distortion.

For distortion, assuming the other aberrations absent,

$$\delta Y' = -\frac{\frac{a_3^2}{2} + a_8}{2a_2} Y^3,$$

$$\delta Z' = 0.$$

This aberration depends only on Y, and is independent of the coordinates η , ζ of the exit pupil. Consequently, the coordinates of the image point are the same for all rays proceeding from the object point. In the absence of diffraction and complete correction of all aberration residuals except distortion, we have, to the degree of approximation contemplated, reproduction of point for point, except that the points in the image will be distorted radially from the

axis, according to the law given above.

Taking into account diffraction, the patterns from selfluminous object point, in the presence of distortion only, will again consist, as in the Airy disk, of central concentration of light surrounded by rings of rapidly diminishing intensity. The position of the central concentration will, of course, depend on the distortion present, but otherwise the light-distribution will be the same as for the aberrationless disk. In the case of the luminous straight line, however, the locus of the central concentrations will be a curve representing the distorted image of the straight line due to the presence of distortion in the system.

Taking the case of the luminous line

$$Y = c,$$

$$X = 0.$$

the reproduction, which should have been Q_1M , becomes the curve $Q_1'T$, defined by the relation $Q_1Q_1'=kO_2Q_1^3$ for positive distortion, where O_2 is the point on the symmetrical axis and the image plane.

The coordinates of Q1' are

$$Y = O_2M + kr^3 \cos \theta,$$

$$Z = MQ_1 + kr^3 \sin \theta.$$

^{*} Lord Rayleigh, "Wave Theory," Encyc. Britannica 9th ed. p. 433.

Taking $O_2M = y$ and $MQ_1 = z$,

$$Y = y + kr^2y = y + ky(y^2 + z^2),$$

$$Z = z + kzr^2 = z + kz(y^2 + z^2),$$

i. e.
$$\frac{\mathbf{Y}}{y} = \frac{\mathbf{Z}}{z},$$

and the equation of the curve Q1'T is

$$\mathbf{Y}^2(\mathbf{Y} - y - ky^3) = ky^3\mathbf{Z}^2,$$

where $y = O_2M$ and constant for the given line.

In like manner the reproduction of X=0, Z=c can be determined; the coordinates of Q_1 are now

$$Y = y + kr^2y,$$

$$Z = z_1 + kr^2 z_1 - kz_1^3,$$

giving as the locus of (Y, Z)

$$Z^{2}(Z-z_{1}) = kY^{2}z_{1}^{3},$$

where z_1 is the height of the reproduction of the point on the Z-axis, which does not suffer distortion since Y=0 for this point. The intensities of light in the reproduction of the first line will be given on the extension of the Airy theory. Thus the intensity at $(Y_1 Z_1)$ will be given by the following integral *:

$$\int_{r=-\infty}^{r=\infty} \left[\int_{r=0}^{r=1} J_0 \left(\frac{r}{f} \sqrt{(Y_1 - Y)^2 + (Z_1 - Z)^2} \right) \pi dr^2 \right]^2 ds,$$

where (Y, Z) satisfies the equation of the curve

$$\mathbf{Y}^{2}(\mathbf{Y}-y-ky^{3})=ky^{3}\mathbf{Z}^{2},$$

and ds is an element of arc at that point. The above formula guides us as to the distribution of light in the distorted diffraction pattern due to the reproduction of the first luminous line, and will consist of a band parallel to the central reproduction along the curve in fig. 1. The corresponding integral for the second curve in fig. 2 will be the same except that (Y, Z) will satisfy the equation

$$Z^{2}(Z-z_{1}) = kY^{2}z_{1}^{3}.$$

* 'Proceedings of the Optical Convention, 1926,' pt. 2, p. 771. [The ratio of semi-aperture to focal length is taken as of the order stronghout.]

The diffraction band will again be parallel to and symmetrical about its parallel to an action of the parallel to a control of the parallel to a contro

metrical about its central curve in fig. 2.

Similarly, the distorted curves could be obtained for other straight lines in the image space, and the corresponding diffraction bands investigated.

Fig. 1.

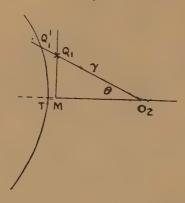
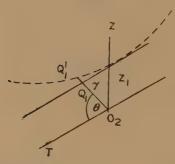


Fig.2.



(b) Spherical Aberration.

Assuming all other aberrations absent except spherical aberration, we have, from equations (4) and (5),

$$\mathfrak{I}Y' = -\frac{1}{2a_2}\eta(\zeta^2 + \eta^2)(2a_5 + 4a_2^3),$$

$$\delta Z' = -\frac{1}{2a_2} \zeta(\zeta^2 + \eta^2) (2a_5 + 4a_2^3),$$

so that

$$\frac{\delta \mathbf{Y}'}{\eta} = \frac{\delta \mathbf{Z}'}{\zeta}.$$

These expressions indicate that the spherical aberration is independent of Y, and that a ray from a point on the axis in the image space is brought to a focus at a point where

$$\mathbf{X}' = -\frac{1}{2a_2} + \alpha,$$

where

$$\frac{\alpha}{X' + \alpha} = \frac{\delta Z}{\zeta} = \frac{\delta Y}{\eta} = -\frac{a_5 + 2a_2^3}{a_2}(\zeta^2 + \eta^2) = -k_2 r^2.$$

Hence α , the longitudinal spherical aberration, can be expanded as a series of even powers of the aperture, verifying a result which has previously been used in determining the distribution of light in the diffraction patterns of a luminous point source in the presence of spherical aberration*. Regarding the luminous line as built up of luminous points, we have, on integration,

$$egin{split} & \left[\int_{-\infty}^{\infty} \left[\int_{0}^{1} \mathrm{J}_{0} \left(rac{r \; \sqrt{\mathrm{Y}'^{2} + \mathrm{Z}'^{2}}}{f}
ight) \cos c_{2} r^{4} \pi \; dr^{2}
ight]^{2} d\mathrm{Z}'
ight] \ & + \int_{-\infty} \left[\int_{0}^{1} \mathrm{J}_{0} \left(rac{r \; \sqrt{\mathrm{Y}'^{2} + \mathrm{Z}'^{2}}}{f}
ight) \sin c_{2} r^{4} \pi \; dr^{2}
ight]^{2} d\mathrm{Z}', \end{split}$$

as representing the value of the intensity at (Y'Z') in the diffraction pattern of a luminous line in the presence of spherical aberration due to a circular aperture.

(c) Curvature and Astigmatism are included in the terms:

$$\begin{split} \delta \mathbf{Y}' &= -\frac{1}{2a_2} \{a_6 + a_7 + \frac{8}{2} a_2 a_3^2\} \, 2 \, \mathbf{Y}^2 \boldsymbol{\eta}, \\ \delta \mathbf{Z}' &= -\frac{1}{2a_2} \{a_7 + \frac{1}{2} a_2 a_3^2\} \, 2 \, \mathbf{Y}^2 \boldsymbol{\zeta}, \end{split}$$

and coma in the remaining terms:

$$\begin{split} \delta \mathbf{Y}' &= -\frac{1}{2a_2} (3\eta^2 + \zeta^2) (a_0 + 2a_3 a_2^2) \mathbf{Y}, \\ \delta \mathbf{Z}' &= -\frac{1}{2a_2} \{ 2\eta \zeta (a_0 + 2a_3 a_2^2) \}. \end{split}$$

* Monthly Notices, R. A. S. lxxxi. No. 8, p. 554.

The curvature and astigmatism residuals may be written as

$$\frac{\delta Y'}{Y^2} = -\eta (3k + k_1),$$
 where $k = a_3^2 - \frac{a_6}{a_2}$, and
$$\frac{\partial Z'}{Y^2} = -\xi (k + k_1),$$
 and $k_1 = \frac{a_6 - 2a_7}{2a_2}$.

If k=0, then, in the first-order image plane, we have

$$\delta \mathbf{Y}' = -k_1 \mathbf{Y}^2 \boldsymbol{\eta},$$

$$\delta \mathbf{Z}' = -k_1 \mathbf{Y}^2 \boldsymbol{\zeta},$$

and as the extreme rays of the pencil satisfy $\zeta^2 + \eta^2 = r^2$, we have

$$(\delta Y')^2 + (\delta Z')^2 = k_1^2 Y^4 r^2,$$

giving a circle about the Gaussian image point, the radius of which increases as Y increases. Hence there is again a point image situated off the ideal image plane at a distance from it which is different for different object points except for the point of the object which lies on the axis with its Y=0. Thus the image of a flat object takes the form of a curved surface of revolution which touches the ideal image plane at its vertex. k_1 is therefore a measure of the curvature of the field:

$$\frac{\delta \mathbf{Y}'}{\eta} = -\mathbf{Y}^2(3k+k_1),$$

$$\frac{\partial \mathbf{Z}'}{\zeta} = -\mathbf{Y}^2(k+k_1),$$
 i. e.
$$\frac{(\delta \mathbf{Y}')^2}{(3k+k_1)^2} + \frac{(\delta \mathbf{Z}')^2}{(k+k_1)^2} = r^2\mathbf{Y}^4,$$
 in ellipse about the Gaussian point

giving an ellipse about the Gaussian point as centre when $k \neq 0$; when $k_1 = 0$, one axis is three times the other. At the point (h, k) farther along the emergent ray and on the plane parallel to and distant α from the ideal focus plane, we have the line joining (η, ζ) and $(Y' + \delta Y', Z' + \delta Z')$ divided

externally in the ratio
$$\frac{\alpha}{X' + \alpha}$$
, $i.e.$
$$X'h = (X' + \alpha)(Y' + \delta Y') - \alpha \eta,$$

$$X'k = (X' + \alpha)(Z' + \delta Z') - \alpha \zeta$$

or
$$X'h = (X' + \alpha) \left[-\frac{a_3}{2a_2} Y - Y^2 \eta (3k + k_1) \right] - \alpha \eta,$$
$$X'k = (X' + \alpha) \left[-Y^2 \zeta (k + k_1) \right] - \alpha \zeta,$$

$$\mathbf{X}'h + (\mathbf{X}' + \alpha)\frac{a_3\mathbf{Y}}{2a_2} = -\eta \left[\alpha + \mathbf{Y}^2(\mathbf{X}' + \alpha)(3k + k_1)\right],$$

$$\mathbf{X}'k = -\zeta \left[(\mathbf{X}' + \alpha)(\mathbf{Y}^2)(k + k_1) + \alpha\right].$$

i.e.

$$\begin{split} \zeta^2 + \eta^2 &= \frac{\left(\mathbf{X}'h + \frac{(\mathbf{X}' + \alpha)a_3\mathbf{Y}}{2a_2}\right)^2}{\{\alpha + \mathbf{Y}^2(\mathbf{X}' + \alpha)(3k + k_1)\}^2} \\ &+ \frac{\mathbf{X}'^2k^2}{\{\mathbf{Y}^2(\mathbf{X}' + \alpha)(k + k_1) + \alpha\}^2} = r^2. \end{split}$$

There are two special cases of this ellipse:

(a) when
$$\alpha + Y^2(X' + \alpha)(3k + k_1) = 0$$
,

(b) ,
$$\alpha + Y^2(X' + \alpha)(k + k_1) = 0.$$

(a) gives

$$\alpha (1 + 3kY^{2} + k_{1}Y^{2}) = \frac{Y^{2}}{2a_{2}} (3k + k_{1}),$$

$$\alpha = \frac{Y^{2}}{2a_{2}} (3k + k_{1}) \quad \text{approx.}$$

(b) gives, similarly,

$$\alpha = \frac{\mathbf{Y}^2}{2a_2}(k+k_1).$$

For these two cases the ellipses reduce to straight lines:

(a) parallel to the Z-axis,

and are known as the focal lines. If k_1 is zero, we see at once that the tangential focal line is three times the distance of the sagittal focal line from the first-order image plane, which now takes the place of the Petzval surface of revolu-When k_t is not zero, the ray intersects—

the Petzval surface where
$$\alpha = \frac{Y^2}{2a_2}k_1$$
,
the sagittal focal line where $\alpha = \frac{Y^2}{2a_2}(k+k_1)$,
and the tangential focal line where $\alpha = \frac{Y^2}{2a_2}(3k+k_1)$.

The theorem is then still true, that within third-order approximation the tangential focal line is three times the distance of the sagittal line from the surface of Petzval.

For the value of α equal to $\frac{1}{2}\frac{Y^2}{2a_2}\left[3k+k_1+k+k_1\right]$ the axes

of the ellipse of section of the pencil are now equal, so that midway between the tangential and focal lines we have the circle of least confusion. The distribution of light in the images of a self-luminous point in the presence of astigmation and curvature of the field have been dealt with for a flat field in the Monthly Notices, R. A. S., and the results can be extended in the usual way by integration to include the luminous line under the same conditions. The integrals for the intensities in the different image planes become very unwieldy, and are not amenable to ready calculation.

(d) Coma.

Lastly, in the presence of coma we have, as was indicated,

$$\delta \mathbf{Y}' = -\frac{1}{2a_2} (3\eta^2 + \zeta^2) \mathbf{Y} (a_9 + 2a_3a_2^2),$$

$$\delta \mathbf{Z}' = -\frac{1}{2a_2} (a_9 + 2a_2^2a_3) 2\eta \zeta. \mathbf{Y}.$$

Writing $\eta = r \cos \theta$,

 $\zeta = r \sin \theta$, and the common constant equal to k,

$$\frac{\delta \mathbf{Y}'}{k\mathbf{Y}} = r^2(2 + \cos 2\theta), \\ \frac{\delta \mathbf{Z}'}{k\mathbf{Y}} = r^2 \sin 2\theta,$$
 giving $\left(\frac{\delta \mathbf{Y}'}{k\mathbf{Y}} - 2r^2\right)^2 + \left(\frac{\delta \mathbf{Z}'}{k\mathbf{Y}}\right)^2 = r^4,$

defining the coma patch in the Gaussian plane.

This patch is seen to consist of a series of circles with centres $(2r^2kY, 0)$ and radii kr^2Y , giving the well-known shape of the coma figure enclosed between two lines at 60°. The square root of the intensity in the central concentration along the Y' axis can be shown to be proportional to the integral

$$\int_0^1 J_0(\mu r + \mu_1 r^3) \pi \, dr^2,$$

 $rdrd\psi$

where μ and μ_1 are constants. The corresponding intensity at any point near the coma axis can be written in the form:

$$I = C^2 + S^2,$$

where C is proportional to

$$\int_{0}^{1} \int_{0}^{2\pi} \cos \left\{ \mu r \cos \left(\phi - \alpha \right) - \mu_{1} r^{3} \sin \phi \right\} d\phi \, r dr,$$
 with $\alpha = \tan^{-1} \frac{\delta Y'}{\delta Z'}$,

and S proportional to

$$\int_0^1 \int_0^{2\pi} \sin\left\{\mu r \cos\left(\phi - \alpha\right) - \mu_1 r^3 \sin\phi\right\} d\phi \, r dr,$$

Writing $\psi = \phi - \alpha$,

$$C = k \int_{0}^{1} \int_{0}^{2\pi} \cos \left\{ \mu r \cos \psi - \mu_{1} r^{3} \sin \left(\psi + \alpha \right) \right\} d\psi r dr$$

$$= k \int_{0}^{1} \int_{0}^{2\pi} \left\{ \cos \left(\mu r \cos \psi \right) \cos \left[\mu_{1} r^{3} \sin \left(\psi + \alpha \right) \right] + \sin \left(\mu r \cos \psi \right) \sin \left[\mu_{1} r^{3} \sin \left(\psi + \alpha \right) \right] \right\} r dr d\psi$$

$$= k \int_{0}^{1} \int_{0}^{2\pi} \left\{ \cos \left(\mu r \cos \psi \right) \left[1 - \frac{\mu_{1}^{2} r^{6} \sin^{2} \psi + \alpha}{2} \dots \right] + \sin \left(\mu r \cos \psi \right) \left[\mu_{1} r^{3} \sin \psi \cos \alpha + \mu_{1} r^{3} \cos \psi \sin \alpha + \dots \right]$$

$$= k \int_0^1 \left\{ \frac{2\pi}{\mu} J_0 + \mu_1 \sin \alpha r^3 \cdot 2\pi J_1 \right\} r dr$$

$$\operatorname{since} \quad \int_0^{2\pi} \sin \left(\mu r \cos \psi \right) \sin \psi \, d\psi = 0,$$

$$\int_0^{2\pi} \sin \left(\mu r \cos \psi \right) \cos \psi \, d\psi = 2\pi J_1$$

$$\begin{split} &= k \, 2\pi \left[\frac{\mathbf{J}_1}{\mu} + \mu_1 \sin \alpha \int r^4 \mathbf{J}_1 \, dr \right] \\ &= k \, 2\pi \left[\frac{\mathbf{J}_1}{\mu} + \frac{\mu_1}{\mu} \sin \alpha \left(\mathbf{J}_2 - \frac{2\mathbf{J}_3}{\mu} \right) \right] \\ &= \frac{2\pi k}{\mu} \left[\mathbf{J}_1 + \mu_1 \sin \alpha \left(\mathbf{J}_2 - \frac{2\mathbf{J}_3}{\mu} \right) \right]. \end{split}$$

452 Aberration Effect on Straight-line Reproduction. Similarly,

$$S = k \int_0^1 \int_0^{2\pi} \sin \left\{ \mu r \cos \psi - \mu_1 r^3 \sin \left(\psi + \alpha \right) \right\} d\psi \, r dr$$

$$= k \int_0^1 \int_0^{2\pi} \left\{ \sin \left(\mu r \cos \psi \right) - \cos \left(\mu r \cos \psi \right) (\mu_1 r^3 \sin \psi \cos \alpha + \mu_1 r^3 \cos \psi \sin \alpha) \right\} d\psi \, r dr$$

$$= 0,$$

since

$$\int_0^{2\pi} \sin(\mu r \cos \psi) d\psi = 0,$$

$$\int_0^{2\pi} \cos(\mu r \cos \psi) \sin \psi d\psi = 0,$$

$$\int_0^{2\pi} \cos(\mu r \cos \psi) \cos \psi d\psi = 0.$$

and

Hence the intensity is proportional to the square of

$$\frac{\mathrm{J}_1(\mu)}{\mu} + \frac{\mu_1 \sin \alpha}{\mu} \left\{ \mathrm{J}_2(\mu) - \frac{2\mathrm{J}_3(\mu)}{\mu} \right\}.$$

Again regarding the linear source as composed of luminous point sources, the illumination at Q'' in the reproduction of the Y-axis is proportional to

$$\int_0^\infty \left[\frac{J_1(\mu)}{\mu} + \frac{\mu_1 \sin \alpha}{\mu} \left\{ J_2(\mu) - \frac{2J_3(\mu)}{\mu} \right\} \right]^2 dY',$$

where Q'' is distant β from the element dY', and β is proportional to μ .

The method has thus been outlined for the Von Seidel aberrations. There remains the investigation due to chromatic aberration, and the effects of the higher-order aberrations on the straight-line reproductions.

Mathematics and Mechanics Dept., The Technical College, Cardiff. Nov. 29, 1928. LII. The Nature and Form of the Contact between two Elastic Bodies pressed together by a Given Force. By H. H. JEFFCOTT*.

1. In many engineering structures and machines various parts made of steel or other elastic material operate in contact with one another, and are pressed together by gravitational or applied forces.

It is sometimes necessary to know the values of the stresses that exist at the point of contact and the dimensions

of the contact area.

The general solution of this problem, as developed by Hertz, leads to somewhat complicated results, involving, in their application to engineering problems, the solution of certain elliptic integrals. The object of this note is to carry the general solution to a further stage in which, with the aid of curves, approximate results may be readily obtained without the necessity of using tables of elliptic functions.

2. The solution of the problem of finding the nature of the contact between two isotropic elastic bodies when pressed together by a given force P has been given by Hertz. (See Love's 'Mathematical Theory of Elasticity,' 3rd edition,

paragraphs 137, 138.)

Hertz supposes the surfaces in contact to be perfectly smooth, so that only a normal pressure acts between the parts in contact, and these parts are assumed to be very small compared with the whole surface. The applied force P is distributed as a pressure over the common area of contact or compressed area. It is required to find the form and dimensions of the compressed area and the pressure distribution over it.

To this end it is necessary to assign such a form to the compressed area, and to obtain such a system of displacements and stresses, that the following requirements are satisfied:—the displacements and stresses shall satisfy the differential equations of equilibrium for elastic bodies; the stresses shall vanish at a great distance from the compressed area; the tangential component of stress shall vanish all over both surfaces; the normal pressure at the surface shall vanish outside the compressed area; within the compressed area the pressure at a point on the surface

^{*} Communicated by the Author.

of one body shall be equal and opposite to the pressure at that point on the surface of the second body; the integral of the pressure taken over the compressed area shall be equal to the total applied force of contact; the distance between the surfaces as altered by the displacements shall vanish within the compressed area and shall exceed zero outside that area.

It follows that the curve of compression which bounds the compressed area is approximately an ellipse. principal radii of curvature of the two bodies at the point of contact be represented by R₁, R₁ and R₂, R₂, and let ω represent the angle between the normal sections of the two surfaces in which the radii of curvature are R₁ and R₂.

Let a and b represent the semi-axes of the ellipse of compression, and let E_1 , σ_1 , E_2 , σ_2 represent Young's Modulus and Poisson's Ratio for the bodies respectively. Let a represent the total compression of the two bodies in the direction of the common normal at the point of contact. This is the distance through which the bodies approach each other under the action of the given force. Also let

$${
m H}\!=\!rac{1-\sigma_1^2}{{
m E}_1}+rac{1-\sigma_2^2}{{
m E}_2},$$
 $k^3\!=\!rac{2\,{
m M}}{3\,{
m PH}}, \quad {
m and} \quad k'\!=\!rac{4a}{3\,{
m PH}}.$

Let M and N be given by equations (1) and (2):

$$M = \frac{1}{R_{1}} + \frac{1}{R_{1'}} + \frac{1}{R_{2}} + \frac{1}{R_{2}^{7}}, \dots (1)$$

$$N^{2} = \left(\frac{1}{R_{1}} - \frac{1}{R_{1'}}\right)^{2} + \left(\frac{1}{R_{2}} - \frac{1}{R_{2'}}\right)^{2} + 2\left(\frac{1}{R_{1}} - \frac{1}{R_{1'}}\right)\left(\frac{1}{R_{2}} - \frac{1}{R_{2'}}\right)\cos 2\omega. \quad (2)$$

Then the results obtained by Hertz reduce to equations (3) to (6):

$$\pi a^{3}(M-N) = 3PH \int_{0}^{\infty} \frac{d\theta}{\left\{\theta \cdot (1+\theta)^{3} \cdot \left(\frac{b^{2}}{a^{2}} + \theta\right)\right\}^{\frac{1}{2}}}, \quad (3)$$

$$\pi b^{3}(M+N) = 3PH \int_{0}^{\infty} \frac{d\theta}{\left\{\theta \cdot (1+\theta)^{3} \cdot \left(\frac{a^{2}}{b^{2}} + \theta\right)\right\}^{\frac{1}{2}}}, \quad (4)$$

Elastic Bodies pressed together by a Given Force. 455

$$4\pi\alpha\alpha = 3PH \int_0^\infty \frac{d\theta}{\left\{\theta \cdot (1+\theta) \cdot \left(\frac{b^2}{\alpha^2} + \theta\right)\right\}^{\frac{1}{2}}}, \quad (5)$$

$$p = \frac{3P}{2\pi ab} \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}, \quad . \quad . \quad . \quad (6)$$

if p represents the pressure at any point x, y of the compressed area.

We observe that the mean pressure $\frac{P}{rah}$ is two-thirds of

the maximum pressure.

When the elastic properties and dimensions of the bodies and the nature of the contact are given, we know

H,
$$R_1$$
, R_1' , R_2 , R_2' , and ω .

Hence M and N are found from equations (1) and (2). Equations (3) and (4) then give a and b, and equation (5) gives the compression α .

3. The elliptic integrals contained in equations (3), (4), and (5) are somewhat troublesome to evaluate, as they require the use of Legendre's or other tables. Accordingly, curves have been drawn here, from which a, b, and α may be approximately derived in terms of M and N, and when a and b have thus been obtained, equation (6) gives the value of the pressure distribution p. These curves have been obtained in the following manner:—

The integral in equation (3), when transformed by the

substitution $\theta = \cot^2 \phi$, reduces to

$$\frac{2}{c^2} \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - c^2 \sin^2 \phi}} - \frac{2}{c^2} \int_0^{\frac{\pi}{2}} \sqrt{1 - c^2 \sin^2 \phi} \cdot d\phi \quad . \quad (7)$$

or $\frac{2}{c^2}(F-E)$, where $c^2=1-\frac{b^2}{a^2}$ and F and E are the

complete elliptic integrals of the first and second kinds.

The integral in equation (4) may be transformed to standard form by putting $\theta = \tan^2 \phi$. It then reduces to

$$\frac{2}{c^{2}}(1-c^{2})^{1/2} \int_{0}^{\frac{\pi}{2}} \sqrt{1-c^{2}\sin^{2}\phi} \cdot d\phi \\
-\frac{2}{c^{2}}(1-c^{2})^{3/2} \int_{0}^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-c^{2}\sin^{2}\phi}} \tag{8}$$

or

$$\frac{2\sqrt{1-c^2}}{c^2} \{ \mathbf{E} - \mathbf{F}(1-c^2) \}.$$

In like manner, on making the substitution $\theta = \cot^2 \phi$, the integral in equation (5) reduces to

$$2\int_{0}^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - c^{2}\sin^{2}\phi}} = 2F. \quad . \quad . \quad . \quad (9)$$

We now have

$$\frac{2a^{3}(M-N)}{3PH} = \frac{4}{\pi c^{2}}(F-E), \quad . \quad . \quad . \quad (10)$$

$$\frac{2b^{3}(M+N)}{3PH} = \frac{4\sqrt{1-c^{2}}}{\pi c^{2}} \{E - F(1-c^{2})\}, \quad (11)$$

$$\frac{4a\alpha}{3\text{PH}} = \frac{2}{\pi}\text{F.} \quad . \quad . \quad . \quad . \quad (12)$$

Dividing equation (11) by equation (10) and reducing, we obtain

$$\frac{N}{M} = \frac{E(2-c^2) - 2F(1-c^2)}{Ec^2}.$$
 (13)

Substituting this value of $\frac{N}{M}$ in (10) and (11) and reducing, we obtain

$$\frac{2 \,\mathrm{M} \, a^3}{3 \,\mathrm{PH}} = \frac{2}{\pi (1 - c^2)} \,\mathrm{E} \quad . \quad . \quad . \quad (14)$$

and

$$\frac{2Mb^3}{3PH} = \frac{2\sqrt{1-c^2}}{\pi}E. \quad . \quad . \quad . \quad (15)$$

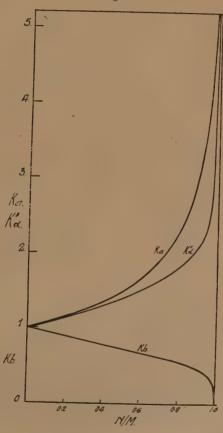
The numerical values of F and E corresponding to various assumed values of c can be obtained directly from the tables of elliptic functions.

Thus from (13) we find $\frac{N}{M}$ in terms of c, and from (14), (15), and (12) we find ka, kb, and $k'\alpha$ also in terms of c. It is convenient to plot ka, kb, and $k'\alpha$ directly in terms of $\frac{N}{M}$. This has been done in Table I. and curves (fig. 1).

TABLE I.

$\frac{\mathbf{M}}{\mathbf{M}}$ 0	•1	•2	•3	•4	•5	•6	.7	·8	•9	1
c 0	•485	647	.749	.822	.875	.917	•948	•972	•989	1
ka1	1.070	1:150	1.241	1.351	1.485	1.661	1.906	2.294	3:085	m
kb 1	936	-877	*823	-769	.718	·664	•608	•544	•461	0
$k'\alpha1$	1.068	1.140	1.216	1.300	1.392	1.503	1.636	1.816	2:100	20

Fig. 1.



- 4. We may illustrate the application of these results by the following examples:—
- (1) A loaded cylindrical steel wheel rests on a straight steel rail of which the upper surface is of circular crosssection. It is required to find the dimensions of the contact.

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2 H

Let the radius of the wheel be represented by r_1 , and let the cross-section at the upper surface of the rail be of radius r_2 .

Then

$$R_1 = r_1$$
, $R_1' = \infty$, $R_2 = \infty$, $R_2' = r_2$, $\omega = 0$.

Equations (1) and (2) give

$$M = \frac{1}{r_1} + \frac{1}{r_2},$$

$$N = \frac{1}{r_1} - \frac{1}{r_2}.$$

Next, let the given dimensions and constants be

$$r_1 = 22$$
 in., $r_2 = 12$ in., $E_1 = E_2 = 3 \times 10^7$ lb. wt. per sq. in.; $\sigma_1 = \sigma_2 = 0.28$; $P = 10,000$ lb. wt.

We find

$$M = 0.129$$
, $N = 0.0379$, $\frac{N}{M} = 0.294$.

The curves give

$$ka = 1.25$$
, $kb = 0.82$, $k'\alpha = 1.205$.

Further, we have

$$H = 6.14 \times 10^{-8}, k = 5.2.$$

Hence

$$a=0.240$$
 in., $b=0.1575$ in.

Also

$$k' = 521$$
 and $\alpha = 0.00231$ in.

Accordingly

$$p_{\text{max}} = 126,000 \text{ lb. wt. per sq. in.}$$

and

$$p_{\text{mean}} = 84,000 \text{ lb. wt. per sq. in.}$$

The ellipse of compression has semi-axes 0.24 in. and 0.157 in.

(2) A spherical steel ball is pressed between the jaws of a measuring machine by a known force. It is required to find the consequent diminution of its diameter.

Let the radius of the ball be denoted by r.

Then

$$M = \frac{2}{r}$$
 and $N = 0$,

since

$$R_1 = R_1' = r$$
 and $R_2 = R_2' = \infty$.

Hence $\frac{N}{M} = 0$, and from the curves ka = kb = 1, k'a = 1.

Suppose we are given that the diameter of the ball is inch, that the force applied by the flat jaws is 10 lb. wt., and that $E_1 = E_2 = 3 \times 10^7$ lb. wt. per sq. in. and $\sigma_1 =$ $\sigma_2 = 0.28$.

Thus $r = \frac{1}{8}$ in., and $H = 6.14 \times 10^{-8}$. k=259, a=b=0.00386 in. Hence

Finally k' = 8391, $\alpha = 0.000119$ in.,

so that the total compression of the diameter due to the contacts is approximately 0.00024 in.

LIII. A Criticism of the Electron Theory of Metals. By H. Monteagle Barlow, A.M.I.E.E., Ph.D.*

Introduction.

N Sommerfeld's recent attempt to reinstate the electron theory of metallic conduction, the physical hypotheses of the classical treatment are employed without alteration t. The only innovation is the supposition that the "electron gas" interpenetrating the atoms of the metal is "degenerate," and therefore does not obey the classical gas laws. The theory assumes a particular form of "degeneration" based on quantum considerations.

After analysing a large number of spectroscopic observations. Pauli concluded that no two electrons in an atom could occupy equivalent orbits, characterized by the same energy values ‡. Fermi, Dirac and others have extended this principle of exclusion to the particles in a gaseous assembly, and shown that for an ideal gas the degree of "degeneration" depends upon the number of particles possessing the same set of quantum numbers §.

Thus, in a "non-degenerate" assembly all the possible stationary states are quite distinct. This extension of Pauli's principle has no apparent physical basis except in so far as a large number of molecules forming a gaseous assembly can be compared with the electrons in a single

^{*} Communicated by the Author.

[†] Sommerfeld, Naturwissenschaften, Oct. 14, p. 63 (1927); and Zeits.

für Phys. xlvii. pp. 1 & 43 (1928).

† W. Pauli, Jr., Zeits. für Phys. xxxi. p. 765 (1925).

† Fermi, Zeits. für Phys. xxxvi. p. 902 (1926); Dirac, Proc. Roy. Soc. A. cxii, p. 661 (1926); Fowler, Proc. Roy. Soc. A, cxiii, p. 432 (1927).

atom. Ordinary gases are supposed to have a very large number of available energy values for their particles at normal temperature and pressure. Hence two or more such particles never want to get into one cell and possess the same "characteristics." The assembly is said to be "non-degenerate" except under high compression and low temperature. On the other hand, Sommerfeld has shown that in a metal the hypothetical electron gas, to which is attributed conduction of heat and electricity, is normally highly "degenerate" in the sense of the Fermi statistics, principally on account of

the very small mass of its particles *.

In working out this idea, Sommerfeld arrived at a number of interesting conclusions, and others have amplified and extended his work †. Recently two valuable papers by Prof. E. H. Hall have been published giving a carefullyconsidered criticism of the theory and the results to which it leads 1. His view is that the new hypothesis regarding the laws governing the electron gas has not been justified, and that the simple classical theory actually gives a better interpretation of some of the facts. One of the consequences of applying the Fermi statistics to the electron gas is to give it a pressure at the zero point amounting to some 100,000 atmospheres, and at normal temperature a still higher value. In the absence of any definite information, the concentration of the free electrons is assumed to be about one per atom under ordinary conditions. This concentration is supposed to vary with temperature and mechanical pressure, but no attempt is made to define these changes, which, Sommerfeld remarks, "can only be explained by a deeper treatment."

Many of the earlier modifications of the classical theory also employ a free electron density which is taken as a function of temperature §, and Prof. Hall, after commenting on the introduction of this idea into the Sommerfeld theory, says: "Since my own theory of metallic conduction etc.

* See also Pauli, Zeits. für Phys. xli. p. 81 (1927).

[†] Eckart, Zeits. für Phys. xlvii. p. 38 (1928); Fowler, Proc. Roy. Soc. A, cxvii. p. 549 (1928); Houston, Zeits. für Phys. xlvii. p. 33 (1928); Borelius, Ann. der Phys. lxxxiv. p. 907 (1927); Frenkel, Zeits. für Phys. xlvii. p. 819 (1928); Nordheim, Zeits. für Phys. xlvii. p. 833 (1928); Houston, Zeits. für Phys. xlviii. p. 449 (1928); Kretschmann, Zeits. für Phys. xlviii. p. 739 (1928); Nordheim, Proc. Roy. Soc. A, cxix. p. 689 (1928); Kretschmann, Zeits. für Phys. xlviii. p. 749 (1928); Kretschmann, Zeits. für Phys. xlviii. (1928); Frenkel & Mirolubow, Zeits. für Phys. xlix. p. 885 (1928); Lennard-Jones & Woods, Proc. Roy. Soc. A, cxx. p. 727 (1928);

Waterman, Phil. Mag. vi. p. 965 (1928).

† Hall, Proc. Nat. Acad. Sci. xiv. pp. 366 & 370 (1928).

§ Hall, Phys. Rev. xxviii. p. 392 (1926) and Proc. Nat. Acad. Sci. (1927); Caswell, Phys. Rev. xiii. p. 386 (1919); Waterman, Phys. Rev. xxII. p. 259 (1923).

has, I believe, sometimes been criticized as involving a considerable number of assumptions of a more or less special character, I am interested in these evidences that the new theory cannot get on without similar inventions. Variation of n (number of free electrons per cub. cm.) with temperature has been one of my cardinal hypotheses." Later he gives a brief review of his dual theory, in which he puts *

$$n = zT^q$$

where z and q are constants and T the absolute temperature. Such variations in the density of the conduction electrons are a logical development from the classical theory, and the position is not materially altered by the introduction of the new statistics.

It is important, therefore, to consider how far experiment confirms this assumption regarding the free electron concentration, and whether we are really justified in treating the ionized negative charges as though they form a gaseous atmosphere interpenetrating the atoms of the metal.

Reasons for Doubting the Validity of the Gas Hypothesis.

Some time ago Lindemann advanced a strong argument, pointing out that the normal forces between the free electrons might be expected to make the assembly behave like a perfect solid rather than a gas †. Even before the electron theory was invented, Maxwell and his contemporaries had come to the conclusion that one of the most fundamental properties of the electric fluid responsible for a current in a conductor was its incompressibility ‡.

The statement that the divergence of the electric force is zero in the interior of a metal, not only expresses the fact that no electricity can be created or destroyed, but also that there can be no accumulation of electricity. This seems to be a natural deduction from the following experimental observations: firstly, that the charge absorbed by a condenser does not depend in any way upon the nature or thickness of the metal plates; and, secondly, that there is no definite indication of any change in the resistance of a conductor when surrounded by an intense electric field.

It is practically certain that the positive nuclei of the atoms of a metal are fixed in position so that, if a charge

* Hall, Proc. Nat. Acad. Sci. xiv. pp. 377 & 802 (1928).

t Maxwell, 'Scientific Papers.'

[†] Lindemann, Phil. Mag. xxix. p. 127 (1915). See also Wolf, Phys. Rev. xxvi. p. 256 (1925) and Hume-Rothery, Phil. Mag. iv. p. 1017 (1927).

distributed throughout the volume is impossible, it must be equally impossible to alter the concentration of the negative particles, including among others the free electrons.

The supposition that the number of free electrons per cub. cm. increases with the temperature is not really consistent with the hypothesis that they form a gaseous atmosphere. For example, it is difficult to explain on that basis how thermal conduction can proceed without building up a considerable charge along the conductor in the direction of the temperature gradient. If the conduction of heat is to be regarded as the result of convection currents in what is equivalent to a gas occluded within a porous body, the simple hypothesis that the free electrons at the hot end of a metal bar are more numerous and have greater kinetic energy than those at the cold end is quite inadequate. Hall found it necessary to assume that the "associated" electrons, which pass from one atomic union to another without sharing the heat energy, also take an important part in the conduction process.

The gas theory is less illuminating when applied to explain the Thomson effect. Without any evidence to show that unneutralized electricity can exist in the interior of a conductor, it is hardly justifiable to suppose that unequal heating sets up an electric field by variations in the number

of free electrons crossing different sections.

Another case in which the gas hypothesis is unsatisfactory is that of an insulated metal bar moving at right angles to its length and perpendicular to a magnetic field. The E.M.F. induced in it produces an accumulation of negative electricity towards one end and a corresponding deficiency towards the other. The force which tends to displace the mobile electricity in the bar acts on the individual electrons, and must operate with equal intensity over the whole cross-section.

If the electric fluid resembles a gas, one would naturally expect the free electrons in the interior of the metal, as well as those at the surface, to drift along the length and build up a charge throughout the volume. The E.M.F. increases with the length of the bar, and the pressure on the free electrons must rise gradually from one end to the other, so that it seems impossible to maintain equilibrium between the forces when unneutralized electricity can only exist on the surface.

A similar inconsistency becomes apparent when considering he electrical effect of rotating a disk at a high speed about its axis. Since an electron has a finite mass, the radial acceleration would be expected to produce an accumulation of negative electricity towards the circumference, and it is clearly quite impossible to balance the centrifugal force on the supposed gaseous atmosphere of free electrons in the interior, if their concentration remains unchanged. In spite of the fact that all these difficulties must be well known, it appears from recent developments that the idea of an electron gas between the atoms of a metal is by no means discredited. Obviously, an incompressible fluid cannot have the properties of a gas, and the all-important question to decide, therefore, is whether the conducting fluid is strictly incompressible or not. The author has attempted to give a conclusive answer to this question by an appeal to direct experiment.

EXPERIMENTAL WORK.

Three different methods of investigation were employed.

1. Experiments on the Electrical Effect of Accelerating a Conductor.

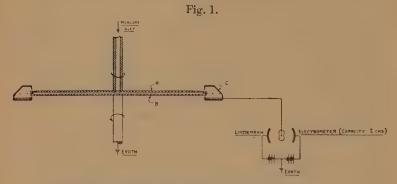
Prof. Tolman and his co-workers have shown that when a conductor, moving with a high velocity in the direction of its length, is suddenly brought to rest, a current is produced during the retardation such as would result from mobile electricity in it having a negative charge *. The conductor formed part of a closed circuit, so that in this case the effect would have been the same whether the conducting fluid were incompressible or not.

In the following experiments (figs. 1 & 2) two copper disks rotating side by side in opposite directions at 3000 R.P.M. were employed to build up a negative charge at the circumference of the combination, which was independent of the effect of the earth's field. Spoonfuls of electricity were carried across from the rim of the two disks to an insulated annular trough of small capacity connected to the needle of a Lindemann electrometer.

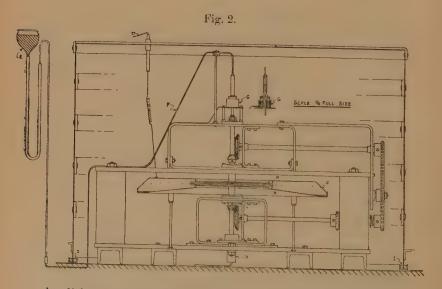
In some of the experiments mercury was fed in at the centre to the small space separating the disks, and the centrifugal force flung it out between the amalgamated surfaces, which it left in the form of globules after assuming their joint potential. Electrification of the mercury by "splashing" was made small by running the apparatus in air at reduced pressure (4 mm.) or in pure hydrogen.

* Tolman & Stewart, Phys. Rev. iii. p. 97 (1916) and Phys. Rev. ix. p. 164 (1917); Tolman, Karrer, & Guernsey, Phys. Rev. xxi. p. 525 (1923); Tolman & Mott-Smith, Phys. Rev. xxviii. p. 794 (1926).

Another method of transferring the electricity from the disks to the surrounding trough without employing mercury was as follows:—A number of small copper blocks, all exactly the same size, were attached to the circumference of



SKETCH SHOWING PRINCIPLE OF MERCURY APPARATUS



the disks by soldering them to fine wires projecting from it. Each disk had the same number of these blocks or bullets, and at a certain speed the centrifugal force was sufficient to break the wires, so that the bullets left the disks to be deposited in the surrounding trough.

The result of these experiments was to show that as much as two cub. cms. of metal flung off the rotating disks did not carry with it more than 10⁻¹¹ coulomb of electricity. According to the simple gas theory this figure is at least 100,000 times too small, showing that any excess electricity on the bullets must have been practically confined to the surface. The very feeble charge that did appear on the trough was negative.

In dealing with the acceleration effects, an experiment was also made on the same principle as that employed by Tolman (figs. 3 & 4). A phosphor-bronze rod; 2 feet 6 inches long, was allowed to fall freely under gravity in the direction of its length through a height of 66 ft., and then brought suddenly to rest at the bottom. The rod formed part of a closed electric circuit, which included a transformer whose secondary winding was connected with a three-valve amplifier with a Paschen high sensitivity galvanometer in the output. The guide-wires for the rod were used to make electrical connexion with it by means of ball contacts amalgamated with mercury.

The current impulses obtained with this apparatus were in the direction anticipated, and their average value was of the right order of magnitude, although the individual figures varied considerably. Spurious effects made the result qualitative rather than quantitative, but the author is not aware of any other experiments that have been made to verify the work of

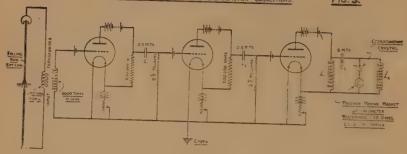
Tolman and his assistants.

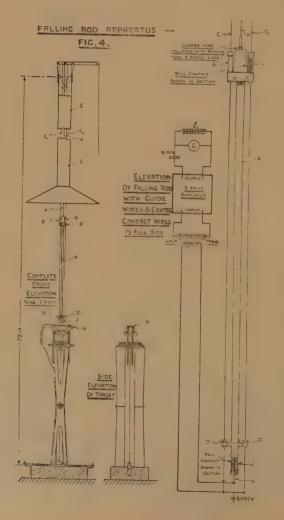
2. Experiments on the Total Displacement of Electricity in an Insulated Conductor subjected to an Induced E.M.F.

The mechanical force on the free electrons due to the radial acceleration in the rotating disks previously described

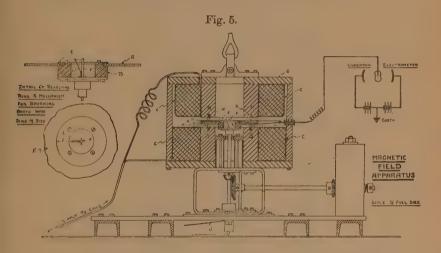
was necessarily small under any conditions.

By employing one disk only with a magnetic field to generate an E.M.F. of about a volt between the centre and circumference, the force tending to squeeze the electrons up towards the rim was considerably increased (fig. 5). If the charge on the disk were not entirely confined to the surface, it seemed necessary to suppose that the unneutralized electricity in the interior must be shielded in some way, so that it added nothing to the induction on the surrounding cylinder. Arrangements were made, therefore, to break the earth connexion at the centre of the disk when rotating, and then to bring the apparatus to rest, the deflexion of the electrometer being carefully observed during the whole process to see if any changes occurred.





Assuming that the electricity squeezed into or out of the disk resided on the surface only, it was found that the potentials calculated from the capacity of the system agreed within 1 per cent. with the actual observations, and these potentials were not reinforced at any time by unneutralized electricity making its appearance from the interior of the conductor. Copper, brass, lead, and graphite disks of the same dimensions all stored the same quantity of electricity, and when the inside of the brass disk was removed, leaving a thin shell so that the capacity remained unchanged while the mass was halved, there was no alteration in the electrometer readings.



3. Experiments on the Resistance of a Charged Conductor.

The free electron theory of conduction leads one to expect that when a conductor is charged, there would be at least a small change in its resistance. An effect of this kind is said to have been observed quite recently *. Another investigator made a similar claim in 1921, but it was afterwards shown to be spurious †. Some earlier experiments by Bose also gave a negative result ‡.

In the following investigation the measurements were made with a simple Wheatstone bridge, the conductor under

^{*} Pierucci, Accad. Lincei Atti. vii. p. 400 (1928).

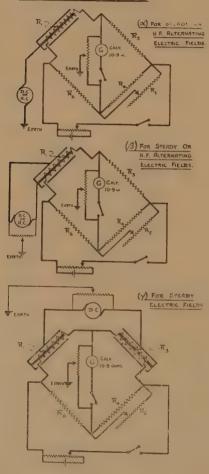
[†] Perkins, Phys. Rev. xviii. p. 131 (1921). Contradicted by Wenner, Forman, & Lindberg, Phys. Rev. xx. p. 589 (1922).

† Described in Sir J. J. Thomson's 'Corpuscular Theory of Matter,'

p. 83 (1907).

observation being charged by electrostatic induction employing either steady electric fields or alternating electric fields of 100,000 frequency (fig. 6). Some very thin films of platinum were prepared by depositing colloidal paint on the

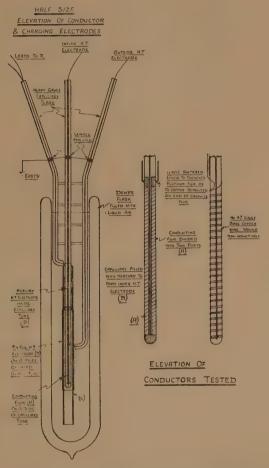
Fig. 6.



outside of a thick glass capillary tube to form a U-shaped conductor (fig. 7). Under a microscope the films appeared quite continuous, although their thickness, in some cases, did not exceed 2 micro-millimetres. Two charging electrodes

were provided so that both sides of the film could be electrified simultaneously, and the whole equipment was immersed in liquid air. Some experiments were also made with graphite films and 47-gauge bare copper wire.

Fig. 7.



In no case did the charging or discharging of the conductor produce an instantaneous and reversible change in the resistance. There was a slight heating effect, but this was easily distinguishable because it took a minute or two to become appreciable. The accuracy of the observations was limited by the tendency by the films to become unstable. In very

intense electric fields irregular resistance changes of a more or less permanent nature appeared, presumably brought about by a re-arrangement of the molecules of the metal.

The most sensitive measurements were made with two platinum films, one in each of the adjacent arms of the

bridge (fig. 6 (γ)).

In this case the estimated thickness of the films was about nine molecules each, and the charge on every square cm. of the surface was equivalent to an excess or deficiency of 4×10^{11} electrons on the inside and 1.7×10^{11} electrons on the outside; yet there was no change in the resistance greater than 0.0012 per cent. The type of change referred to here is one which appears as soon as the charge is produced, and disappears immediately the charge is removed.

Conclusions.

The foregoing experiments prove conclusively that the assembly of free electrons in the interior of a conductor invariably behaves like a perfectly incompressible fluid, and consequently in no way resembles a gas. The fundamental hypothesis of the Sommerfeld theory of conduction cannot, therefore, be maintained.

Finally, the author desires particularly to express his thanks to Prof. J. A. Fleming, M.A., D.Eng., D.Sc., F.R.S., for giving so freely of his time and experience in the development of these investigations, and also gratefully to acknowledge the facilities granted by Prof. Fleming and Prof. Clinton for carrying out the experimental work.

University of London, University College, Gower Street, W.C. 1. December 1928.

LIV. The Phenomena of Projected Electrons.

To the Editors of the Philosophical Magazine.

GENTLEMEN,-

I has been suggested to me that I should state more definitely the conclusions which can be deduced from a recent note * of mine with regard to the Einstein theory of Relativity.

* "A Note on the Problem of the 'Mass' of a Moving Electron" (Phil. Mag. vol. v. p. 636, March 1928).

In the first place, the argument refers to the form of the electric lines of force near to a plane conductor, when an electron is moving away from that boundary; while the application refers to the motion of the electron, as it emerges from the slit of a condenser formed of two conducting plates (as in the Bucherer type of experiment). But, although the use of the plane boundary would give a poor estimate of the actual effects of the conductors, yet it illustrates the kind of disturbance of the field which must take place; so that near to the condenser, the lines of force must deviate from perfect symmetry, and it is therefore evident that the electrons are retarded during their flight from the opening or slit of the condenser into that part of the space where the velocity can be regarded as finally steady. It would be almost impossible to calculate the deviation that takes place, during this retardation, with any degree of accuracy; but, by means of fairly simple considerations, I have convinced myself that the deviation near the slit is small compared with the probable errors of observation. Thus, the deviations observed by Bucherer (and by later experimenters) can be calculated with quite sufficient accuracy by means of the Lorentz-formula; and this agrees with the numerical results found by Lorentz himself, and in similar calculations made with reference to other experiments (of the same general type as those of Bucherer).

Secondly, working on the basis of the Lorentz-formula, in recent experiments it appears that the average speed v of the electrons must be such that $\beta (=v/c)$ is at least 9; in some cases a value as high as 95 has been recorded. Now, in view of the retardation already mentioned, it is clear that the velocity with which the charged particle leaves the slit must be greater than that with which the steady path is described finally. Taking these two points into consideration it is not unreasonable to suppose that at the slit itself, v = c. In verbal discussions of these ideas, the usual objection to the possibility of motion at speed v = c is raised, that this suggestion involves infinite energy, in view of Lorentz's formula *

$$\frac{2}{3}\frac{e^2}{a}\frac{1}{\sqrt{1-\beta^2}}$$

where, as above, we write $\beta = v/c$.

^{*} The factor 2/3 corresponds to a surface-density; it is changed to 4/5 for a volume-density. See, for instance, G. A. Schott, 'Electromagnetic Radiation,' Arts. 143 and 240.

To discuss this argument, it is desirable to recall the method used by Lorentz in obtaining his formula; the electron is reduced to rest by applying the Lorentz-transformation; and the transformed field is assumed to be of the simple radial type, associated in electrostatics with a uniform surface-density on a spherical conductor. That is, the transformed field is supposed to extend to infinity; this assumption will not be likely to affect the character of the field near to the electron itself, but it has considerable influence on the result found for the energy. In all known wave-solutions*, there is a pulse or skin (to use Heaviside's own term), which travels with speed c; this pulse separates two distinct fields from each other, and the energy in the pulse is of the same order as that in the interior †.

Thus Lorentz's formula implies that no account is taken of the outer boundary of the field; and so far, any attempt to extend his method to take such account has failed. But we may use the solution roughly indicated in my former note; and this gives some idea of the magnitudes involved. Supposing that the electron can be maintained at a velocity equal to c, the pulse will always contain the electron, the field beyond the pulse being either simply electrostatic, or else zero; and the lines of electric force will be condensed into the thin skin (its thickness will be equal to the diameter of the electron). The formula for the energy which corre-

sponds to Lorentz's quoted above is then

$$\frac{e^2}{a} \left\{ \log \left(\frac{\mathbf{R}}{a} \right) + 1 \right\},\,$$

where R is the radius of the central sphere of the pulse.

Under the conditions of the Bucherer experiment R cannot exceed the radius of the cylinder on which the photographic plates are bent, so that R may be supposed of the order of 10 to 20 cm.; and for β -particles the radius α will be of the order 10^{-13} cm., which makes the bracket in the last formula about equal to 34. This is roughly twice the numerical factor derived from the Lorentz formula, if we suppose β = 999. It would seem clear therefore that the statement that v=c involves infinite energy is misleading.

Thirdly, having accepted as a working hypothesis that the electrons leave the slit at a speed v = c, we naturally inquire whether such a speed can be maintained between the plates of

^{*} See, for instance, G. A. Schott, l. c. ch. v. (see pp. 75, 76, for typical diagrams); and Heaviside, 'Electromagnetic Theory,' vol. i. pp. 55-62, and vol. iii.

† Heavide, 'Electromagnetic Theory,' vol. iii. pp. 127-134.

the condenser itself. Now, the extremely simple solution briefly stated in my note * shows that, at speed v = c, the lines of force remain in planes perpendicular to the line of flight, and thus no retardation occurs in the direction of the line of flight. As regards the transverse electric force, this is neutralized (as in the usual version of the theory) by the crossed magnetic field. It follows that between the plates of the condenser, a steady motion, parallel to the plates, is possible at the special speed v = c. The objection may be urged that it is unlikely that every charged particle will be ejected from a radioactive substance at the precise speed v = c. Now, if at the instant of ejection v < c (since there is no source of subsequent acceleration), the speed will never attain the value v=c; accordingly, we must suppose that, initially, $v \ge c$. As regards the case v > c, the only detailed solutions at present known are due to Heaviside †; these refer to charged particles moving in an unlimited space, and they indicate that the lines of force, as they leave the electron, must be bent back so as to lie within a cone of vertical angle 2α , where $\sin \alpha = c/v$. At a later stage the lines of force must turn so as to cut the plates of the condenser orthogonally, but the resultant force on the charged particle is due to the direction in which the lines of force leave the particle; thus there must be a marked retardation at speeds greater than c. It does not appear to be possible to make any exact determination of the distance through which a particle would travel at these high speeds; but, if the formula for the retardation, originally found by Sommerfeld ‡, is used, the distance cannot exceed a moderate multiple of the radius a; for example, if the speed is $v = \frac{3}{5}c$, the Sommerfeld formula

* To all intents and purposes this result is due to Heaviside; although I have not noticed that he ever actually gave the particular interpretation

which is stated in my note.

† See, for instance, his 'Electromagnetic Theory,' vol. iii. § 465 and §§ 497-498; some examples with v=c are given in §§ 499-500. I have succeeded in obtaining some extensions of these solutions which throw further light on the character of the fields in the pulse; but for our immediate purposes the results given by Heaviside will be sufficient.

† Calculated by Sommerfeld ('Göttingen Nachrichten,' 1905) for a

particle moving in unlimited space as equal to $\frac{9}{4}\frac{e^2}{a^2}\cot^2\alpha$; this is

obtained by more elaborate methods, but these would not (as far as I can see) be successful in taking any account of the plates of the condenser.

It may not be out of place to refer to the fact that the existence of such cones has been detected experimentally in photographing the flight of a bullet at speeds greater than that of sound. See, for instance, P. P. Quayle, Franklin Institute, vol. exciii. p. 627 (1922).

for retardation, when multiplied by 12a, gives an energy nearly equal to that found in our previous calculation (of the energy of an electron emerging from the slit). Consequently, if a charged particle should be ejected from the radioactive substance at a speed v > c, the particle will be retarded at once, and the speed will accordingly be reduced until the value v = c is reached, after which the speed can be maintained at this constant value, between the plates of the condenser, without assuming that any external force is applied.

The question may now be raised, as to the energy required here, if the electron moves with speed c between the plates of the condenser. An exact solution is then possible for the case when the electron is supposed to remain spherical, and to move parallel to a plane conductor; and for the present problem an approximation is possible, in consequence of the minuteness of a compared with the other dimensions; neglecting terms of the order $(a/h)^2$, it appears that the formula above will represent the energy, provided that R is replaced by

 $\mathbf{R} = \frac{h}{\pi} \sin\left(\frac{\pi b}{h}\right),\,$

where h is the distance between the plates of the condenser, and b is the distance of the centre of the electron from the nearer plate. Thus, if h is large compared with b, we can use R = b, which agrees (to the same order) with the value deduced from the more complete solution available when there is only one plane boundary. It follows that the energy required is of the same order as in the estimate made earlier (for an electron emerging from the slit of the condenser).

Accordingly, I put forward the suggestion that the high-speed particles may start originally with v > c; they will then be retarded until the speed v = c is attained; afterwards they will move steadily (without external force) parallel to the plates of the condenser; when the slit of the condenser is passed, the particles will be again retarded, in a manner which depends on how the lines of force bend backwards (so as to cut the faces of the condenser orthogonally). The part of the later flight which produces the observed deviations will be described practically under the conditions of the Lorentz-formula (a steady value of v < c).

The question will now suggest itself:—How do these conclusions affect the theory of Relativity? Regarded as a branch of modern Pure Mathematics, it is evident that the theory is entirely unaffected by any questions of Physics; just as the existence of Weierstrass's and other non-

differentiable functions is not affected by the fact that no one has yet made a graph to which a tangent cannot be drawn. And, so far as the supporters of Relativity are content to regard their theory as having enlarged the region of applications of the Absolute Calculus of Levi-Civitá (and of other Italian geometers), there is nothing to prevent them from pushing their analysis to the uttermost limits of refinement.

But, when we consider the question as to how far the theory of Relativity has any direct bearing on Physics, it seems to me that, so far, we can only regard the theory as having grown out of Lorentz's simplified method of calculating the deviations observed by Bucherer. Thus I suggest that its importance consists mainly in having provided the most convenient way of carrying out the necessary analysis. To take an illustration from a cognate subject, it would appear to be on much the same footing as the Euclidean theory of parallels, which may be regarded as the simplest theory consistent with observations. It is known that other investigators have established more complicated non-Euclidean geometries; but, within the limits of the best astronomical work, nothing has yet been observed to distinguish between the three kinds of geometry (hyperbolic, elliptic, and ordinary Euclidean). Thus, for working purposes we are justified in using only Euclidean geometry (as the simplest of the three); but certainly this does not enable us to assert that no distinction will ever be discovered with further refinements of instruments. In the same way, the mere fact that the Lorentz-formula gives the most elegant method of dealing with Bucherer's experiments is a very slight basis for the assumption that charged particles cannot travel at a speed v = c or v > c.

Naturally, the objection will now be urged that the theory of Relativity has been applied with success to other problems. Let us consider first the astronomical problem of the motion of the perihelion of Mercury. In actual fact the value of $\beta = v/c$ is of the order 10^{-4} in this case; and thus, since

$$1/\sqrt{(1-\beta^2)} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \dots,$$

it is evident that, even with the best astronomical instruments, no distinction can be made here between the Lorentz-formulæ and the formulæ given earlier by Larmor *, who expanded the electromagnetic equations in powers of β , retaining terms of the order β^2 . Thus here, again, the

^{*} In his Adams Prize Essay, "Æther and Matter," ch. xi.

preference for the Lorentz-formula rests on an appeal to Pure Mathematics; for it has been found * that the resulting analysis is made simpler and more elegant by using Lorentz's methods. The same argument applies (with even more force) to the conclusions drawn from the Michelson-Morley experiment: for in that the value of β is less than for Mercury, so that the distinction between $1/\sqrt{(1-\beta^2)}$ and $1 + \frac{1}{3}\beta^2$ cannot be observed with our present instruments.

The last "proof" of the theory of Relativity, and one which is often assumed to be conclusive, is based on observations of the bending of light-rays, produced by the sun's gravitation; the results are well known from the reports of recent eclipse-expeditions. Such observations certainly justify the inference that the phenomena of light and of gravitation have an intimate connexion; but the exact formulæ, originally put forward as representing the consequences of Einstein's theory, have been repeatedly criticized by Sir Joseph Larmor. An adequate summary of his arguments would occupy too much space here; but we may refer to his address to the Royal Society of Edinburgh † (on the presentation of the James Scott Prize) for a synopsis of his reversion to common space and time (from the relativist point of view).

Writing from a mathematical standpoint I should like to acknowledge the debt which all students of the mathematical theory of pulses and of electromagnetic waves owe to the brilliantly successful experiments of the past 30 years; without such experimental knowledge the best attempts at mathematical solutions would be valueless, owing to lack of any knowledge as to the orders of magnitude involved 1.

I have the honour to be. Gentlemen.

Cambridge. 12 May, 1928. Your obedient servant, T. J. PA. BROMWICH.

* See, for instance, the account of the Dynamics of a moving electron in G. A. Schott's Adams Prize Essay ("Electromagnetic Radiation," Cambridge, 1912, pp. 257-325).

† Larmor, Proc. Roy. Soc. Edinb. vol. xlvii. p. 307 (1927); see, in

particular, pp. 320-325, for considerations dealing with the gravitational

† I take the opportunity of expressing my personal indebtedness to various friends who have put at my service their knowledge of the physical circumstances of the Bucherer experiments and of the results derived from these and other works connected with them.

For some instructive remarks on the information to be derived from experiments and from mathematics, the reader may consult Heaviside's 'Electromagnetic Theory,' vol. iii. pp. 472-478, and pp. 131-134 the latter containing the first attempt to connect numerically the "mass" and charge of these particles.

LV. A New Transformation in Alternating Current Theory, with an Application to the Theory of Audition. By Balth. VAN DER POL, D.Sc.*

[Plate VII.]

IN ordinary linear alternating current theory an inductive reactance is usually represented by +jwL, while a

capacitive reactance is denoted by $-rac{j}{\omega \mathrm{C}}$. Therefore the

imaginary part of an impedance may have a positive as well as a negative sign. As long as positive resistances only were known, the real part of an impedance was always positive (independent of the constitution of the network considered). But with the advent of negative resistances (such, e. g., as the arc, the dynatron, or the triode with retroaction), the real part of an impedance, like the imaginary part, may have

both signs.

The transformations to be considered here (and which may be called *j-transformations*) consist of multiplying all complex impedances of a network by j, j^2, j^3 , and j^4 respectively, where $j = \sqrt{-1}$. Though mathematically these transformations obviously only represent a turning of the impedance vectors by angles $\pi/2$, π , $3/2\pi$, and 2π respectively, the physical interpretation of these transformations may lead to some new circuits or to, at first sight, unexpected relations

between circuits with well-known properties.

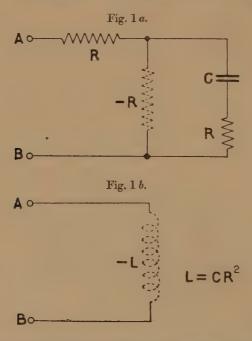
Obviously the j^4 -transformation transforms the circuit into itself, and therefore does not yield anything new. On the other hand, the j^2 -transformation implies the multiplication of any circuit element with -1. This means that in the network considered every positive resistance is to be replaced by a negative resistance of the same absolute amount and vice versa, and also every positive capacity by a negative capacity and vice versa. With this physical interpretation of the j^2 -transformation any network thus transformed retains its impedances (but multiplied by -1) for all frequencies. The necessity of constructing negative inductances and negative capacities does not (at least on paper) yield any difficulties now that negative resistances are at hand; for the equivalent network of fig. 1 a is fig. 1 b, as can be easily verified.

We can therefore, with the aid of two positive and one negative resistance and a capacity, construct a negative

^{*} Communicated by the Author. Paper read at the U.R.S I. Meeting, Brussels, Sept. 1928.

inductance which is independent of frequency. The circuit of fig. 1 a thus behaves exactly as if between the terminals A-B a negative inductance were inserted of amount $L = CR^2$.

Similarly, a negative capacity (again independent of frequency) may be obtained as in fig. 2, which speaks for itself*. Thus the network of fig. 2 a is equivalent to a negative capacity of amount $C = LR^{-2}$ (fig. 2 b), as is easily verified. Of course, instabilities often arise in circuit comprising, inter alia, negative resistances. The stability of such



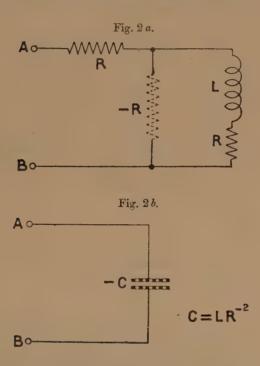
a circuit can, however, be calculated. We therefore imagine with Barkhausen (*Phys. Zschr.* xxvii. p. 43, 1926) a small inductance to be inserted in series with an arc, and a small capacity to be shunted to a dynatron or triode. This system can then further be investigated with the aid of the Hurwitz determinants, giving the conditions that the real part of all the roots of an equation are negative. (I found that these determinants can be considerably simplified, and hope to report on this subject shortly.)

^{*} See Bartlett, Journal of Inst. of Elect. Eng. (London), lxv. p. 373 (1927); also British Patent 278036.

Coming now to the *j-transformation* and the j^3 -transformation, it will be clear that the latter can be derived from the former by a j^2 -transformation, so that we can further limit

ourselves to consider the j-transformation only.

As the reactance of a self-inductance and of a capacity depends upon the angular frequency ω of the impressed E.M.F., the j-transformation can be used for one given frequency at a time only, so that when a different frequency is considered, the physical numerical result of the j-trans-



formation is also different. In applying the j-transformation, every positive resistance R in the network, through the multiplication by j, becomes jR, and this has to be interpreted physically as $j\omega L'$, i. e. every positive resistance R is transformed into an inductance L', of amount $L' = R\omega^{-1}$. Similarly, every negative resistance —R has to be replaced by a capacity C' such that $('' = (\omega R)^{-1})$. Further, every impedance $j\omega L$ becomes $jj\omega L = -\omega L = -R'$, i. e. every inductance L has to be replaced by a negative resistance —R' of an amount $R' = (\omega L)^{-1}$. Finally, every capacitive reactance $1/j\omega C$ becomes $j/j\omega C = (\omega C)^{-1}$. This expression

is real and positive; hence every capacity (of value C) has to be replaced by a resistance R', where $R' = (\omega C)^{-1}$.

It will be clear that in ordinary vectorial representation the j-transformation simply means revolving a vector diagram as a whole over 90°, and then to inquire into the physical meaning of the revolved diagram.

The j-transformation could obviously be extended to transformations such as a $j^{\frac{1}{2}}$ -transformation, meaning a rotation over an angle $\pi/4$. However, we will limit ourselves here

to integral powers of j only.

Consider, e.g., a series connexion of L, C, $+r_1$, and $-r_2$. Upon this linear system let an E.M.F. be impressed of angular frequency ω . By varying the L or the C, we may bring the system into "reactance-resonance." When, as in a wireless receiver with retroaction, we thereupon make r_2 as near as possible equal to r_1 (i.e. adjust the retroaction to the critical point), we do the same thing over again, but only after one j-transformation, and one could say, in a broad sense, that adjusting the total resistance near zero value means bringing the system into "resistance-resonance."

We will now consider some simple a.c. circuits having special properties, and inquire as to what becomes of them

after a j-transformation.

Example 1.

Consider fig. 3 a, representing a well-known circuit, such as is often used, e.g., in a triode amplifier. This circuit has the property that for resonance, i.e. for

$$j\omega \mathbf{L} = -\frac{1}{j\omega \mathbf{C}}, \quad \dots \quad \dots \quad (1)$$

the equivalent impedance Z_{AB} between the points A-B is

$$Z_{AB} = \frac{\frac{L}{C} + r^2}{2r}, \quad . \quad . \quad . \quad . \quad (2)$$

and is therefore real, i. e. a pure resistance. After one j-transformation, fig. 3a is changed into fig. 3b, such that

$$j\omega \mathbf{L} = -r_1',$$

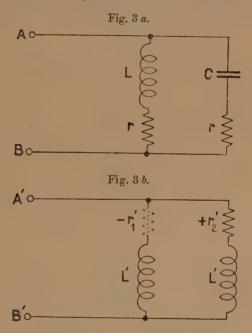
 $\frac{1}{j\omega \mathbf{C}} = +r_2',$
 $r = j\omega \mathbf{L}'.$

Thus the condition (1) becomes

$$r_1' = r_2'; \ldots \ldots \ldots (1')$$

i. e. in the derived circuit the equality of the moduli of the two resistances corresponds to the resonance condition of the original circuit. And, further, the special property (2) of the original circuit becomes, after our j-transformation,

$$Z'_{A'B'} = \frac{-r_1'r_2' - \omega^2 L'^2}{2j\omega L'} = j\omega L' \cdot \frac{\omega^2 L'^2 + r'^2}{2\omega^2 L'^2};$$
 (2')



or, in other words, the system of fig. 3 b behaves as a pure inductance when the numerical values of $-r_1$ and r_3 are made equal.

Example 2.

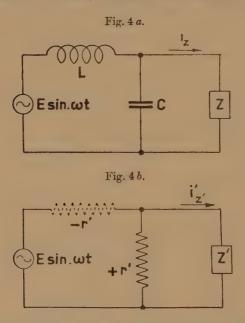
The circuit of fig. 4a has the property that when $j\omega L = -\frac{1}{j\omega C}$, i. e. in resonance, the current i_Z through the arbitrary impedance Z is independent of this impedance, viz.

$$i_{\rm Z} = \frac{\rm E}{j\omega \rm L}$$
. (3)

After the application of the j-transformation, fig. 4a becomes fig. 4b. Here, again, the resonance condition of fig. 4a is changed into the equality of the numerical values of the two resistances -r' and r' in fig. 4b. The arbitrary impedance of fig. 4a is transformed into another arbitrary impedance Z' of fig. 4b. Again (3) changes into

$$i_{\mathbf{Z'}} = -\frac{\mathbf{E}}{r}, \quad \dots \quad (3')$$

or in words: the current through Z' is independent upon the impedance Z' itself, so that when, instead of $E \sin \omega t$,



an E.M.F. is applied of arbitrary wave-form, the current through Z' is an exact image in amplitudes, as well as in phases, of the applied E.M.F.*

Example 3.

Let a given current i be flowing between the terminals A and B of fig. 5 a. It is easy to show that for resonance the P.D. $\mathbf{E}_{\mathbf{Z}}$ developing across the impedance Z will be

$$\mathbf{E}_{\mathbf{Z}} = \frac{i}{j\boldsymbol{\omega}\mathbf{C}},$$

* This property of fig. 4 b was already derived by Bartlett, l. c.

and therefore does not depend upon the value or constitution of Z.

The j-transformation of fig. 5a is given by fig. 5b. If in the latter the negative resistance -r' is numerically made equal to the positive resistance r', then the P.D. developing across the arbitrary impedance Z' is an exact image of the current i' entering the system, independent of its wave-form.

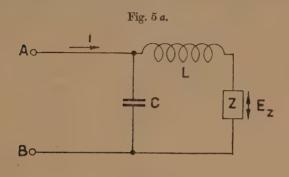
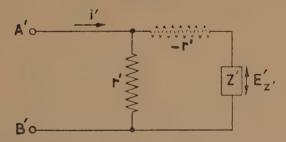


Fig. 5 b.



Example 4 (application to the theory of audition).

Consider the circuit of fig. 6 a. For an impressed E.M.F. of a frequency ω given by

$$|\omega^2 = \frac{1}{2LC} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the impedance ZAB between the terminals AB is given by

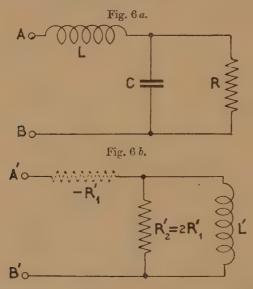
$$Z_{AB} = j\omega L \cdot \frac{1 - j\omega CR}{1 + j\omega CR} = j\omega L \cdot e^{-2j\theta}, \quad . \quad . \quad (5)$$

where

We therefore see that for the specific frequency given by (4) the modulus of the impedance between the points A and B of fig. 6 a does not vary with the value of R. If, thus, an alternating E.M.F. of constant amplitude and of frequency $\omega^2 = (2LC)^{-1}$ is maintained between the terminals A-B, the amplitude of the current through the L-branch is unaffected by the value of R, but its phase is. This is a very valuable property for measurement purposes, as we can change the phase over 180° without affecting the amplitude.

When we now apply again the j-transformation to the

system of fig. 6 a, fig. 6 b is obtained.



The frequency condition (4) of the former circuit, which can be written

$$j\omega \mathbf{L} = \frac{-1}{2j\omega \mathbf{C}},$$

now becomes

$$R_2' = 2R_1'$$

and does not depend upon the frequency. The impedance $Z'_{AB'}$ of fig. 6 b now becomes

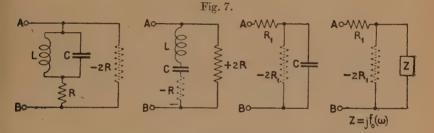
$$Z'_{A'B'} = -R_1' \cdot \frac{R_2' - j\omega L'}{R_2' + j\omega L'} = -R_1' \cdot e^{-2j\theta_i}, \quad . \quad (5')$$

where

$$\tan \theta' = \frac{\omega L'}{R_2'}. \qquad (6)$$

Our circuit of fig. 6 b has therefore the following properties: the modulus of the impedance of the network between the points A' and B' is the same for all frequencies, and does not depend upon the value of L'. The phase, however, does depend upon the value of L'. If we therefore apply an E.M.F. of any complicated wave-form to the terminals A'B', all the amplitudes of the various harmonic components of the current entering at A' will be exactly proportional to the corresponding amplitudes of the harmonics of the E.M.F. I he phases of the currents, however, are shifted relatively to the phases of the E.M.F. and by an amount depending upon the frequency.

The limiting cases can be recognized directly. For a very low frequency the L' branch forms a short circuit in parallel to the R_2 ' branch. Hence, for a very low frequency the impedance $Z'_{A'B'}$ approaches $-R_1$ '. For a very high



frequency, on the other hand, the branch L' (in parallel to the R_2 ' branch) can be ignored, thus leaving a total impedance $Z'_{A'B'} = -R_1' + R_2' = -R_1' + 2R_1' = +R_1'$. Thus the total possible phase-change is 180°.

Several circuits possessing the above properties can be designed. Some other instances are given in fig. 7. (By $Z=j_{f_0}(\omega)$ is meant any impedance consisting of inductances and capacities only.)

For all those circuits having the above property the

impedance Z can be expressed as

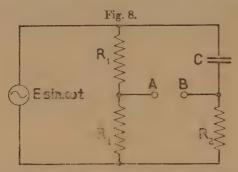
$$\mathbf{Z} = \mathbf{R} \cdot \frac{f_e(\omega) - j f_0(\omega)}{f_e(\omega) + j f_0(\omega)} = \mathbf{R} \cdot e^{-2j\theta},$$

where $f_e(\omega)$ and $f_0(\omega)$ represent an even and an odd function of ω respectively, and where

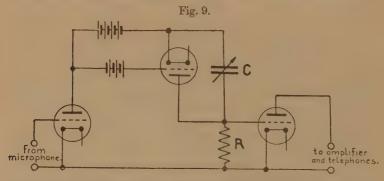
$$\tan\theta = \frac{f_0(\omega)}{f_s(\omega)},$$

and R represents an ohmic resistance.

It may be remarked that the circuits considered are quite different from the circuit of fig. 8, described by Möller *, where the amplitude of the potential between A, B is independent of the frequency of the source E $\sin \omega t$, while the phase is dependent upon it. However, this is only true as long as the internal resistance of the source E $\sin \omega t$ is zero, which is difficult to realize in practice with triodes.



The property described of leaving the amplitudes of all the composing harmonics untouched but only changing their phases gives us a means of verifying experimentally the well-known acoustical law of Ohm, which states that our ear perceives from a complicated sound the amplitudes of the various components only, but that it cannot recognize



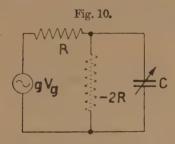
the phases of these components. Therefore arbitrary changes made in the phases of the components do not vary our perception of a sound so long as the amplitudes are left unaltered.

Now, in order to test this law the circuit of fig. 9 was used.

* H. G. Möller, 'Schwingungsaufgaben,' p. 43 (Hirzel: Leipzig, 1928).

The first triode acts as an amplifier, while the second triode is used as a dynatron providing the negative resistance. Essentially this circuit is equivalent to the circuit of fig. 10. The current through R represents an exact amplitude image (not phase image) of the grid potential of the first triode. The potential variations over R are the grid potential variations of the third triode.

If the circuit of fig. 9 is properly adjusted, the amplitudes of the various components of the sound spoken in the microphone are not affected by a change of C, but their phases are. If, now, one listened to a loudspeaker connected to the output side of the circuit of fig. 9 while someone else spoke into a microphone set up in a different room, not the slightest difference would be observed after a pronounced change (up to 180° and, with a L inserted in series with the C, even 360°) of the phases of the components. This fact was



verified by several persons, so that the above law of Ohm was found true for phase changes up to 360°. Obviously the microphone and the amplifier produced some phase changes already. The fact, however, that the phases of the components could be advanced as well as retarded, makes the above conclusion justified.

For non-periodic sounds, such, e.g., as the spoken language, phase retardation may be conceived of any amount extending beyond 360°. For instance, at the receiving end of a long telephone cable the time of arrival of the higher speech components may be considerably different from that of the lower components; this relative retardation may obviously amount to several periods. One cannot expect the ear to be absolutely insensitive to such long relative retardations, and practice with cables bears out this fact plainly.

For periodic sounds (such as vowels) and normal amplitudes, however, the above experiments conclusively confirm the classical acoustical law of Ohm, which was also found to be valid for aperiodic sounds (the spoken word) as long

as the relative phase retardation between the high and the low components was not greater than 360° of the high

components.

Finally, oscillograms were taken of various vowels with and without changes of the relative phases, and the curves for the vowel a (the pronunciation in the Dutch language of this vowel is quite near that of the German language) is given in fig. 11 (Pl. VII.) where the lower oscillogram gives the undistorted vowel while the top one is phase-distorted (fundamental frequency = 140 sec. -1). It is curious to note that, though the wave-form of the two oscillograms is quite different, there is not the slightest difference audible. This oscillogram was taken by Mr. van der Mark of this laboratory, who in another article describes the technique of these experiments (where also further instances of vowel oscillograms will be found).

Natuurkundig Laboratorium der N. V. Philips's Gloeilampenfabrieken, Eindhoven.

LVI. Concerning the Artificial Preparation of Diamonds.
By Dr. Luciano Sesta *.

[Plate VIII.]

THE problem of the artificial reproduction of diamonds is already an old one, and many experimenters have attacked it with the hope of snatching even this secret away from Nature.

Since 1694-95, in Florence, when Averani and Targioni proved that diamond burning at a temperature of 700-800° C. in an atmosphere of oxygen gives as a product of the combustion carbonic acid gas (CO₂), it has by various means been tried to obtain it in the laboratory.

Of all the attempts the most successful was that by Moissan, who, after patient and careful study, succeeded in 1896—obtaining minute examples of artificial diamonds.

The conclusions at which he arrived are as follows +:-

(1) Graphite is the variety of carbon stable at a high temperature and under ordinary pressure.

^{*} Communicated by Prof. M. La Rosa. † Ann. de Chimie et de Phys. 1896.

(2) Carbon passes from the solid form to the gaseous, that is, it sublimes, and when condensed these vapours give graphite.

(3) Diamonds can only be produced under very great

Guided by these results, he made his famous experiment with a result that has been retained positive. Later, other experimenters, such as Majorana, Hasslinger, Fischer, and others, undertook these studies following the directions of Moissan, but did not obtain better results.

In 1909, Prof. M. La Rosa, placing under critical examination the experiences of Moissan, saw a newer and clearer, and one might add more elegant, path to follow

than that pursued by Moissan and his followers.

Prof. La Rosa has observed first of all that the ultimate deductions of Moissan go beyond the bounds of his experi-

ences; in fact, he expresses himself thus *:-

".... The experiment had only showed that, at the temperature of the voltaic arc and that of the electric furnace carbon sublimes without presenting traces of fusion: and that the fusion was effected, probably, under the very high pressure resulting from the solidification of cast iron: hence it was only possible to affirm that carbon does not fuse at the temperature of the electric furnace, and that furthermore it is one of those bodies that assume under great pressure a lower point of fusion. In order to conclude that carbon does not melt under ordinary pressure, it is still necessary to prove that the vapour-pressure of carbon exceeds atmospheric pressure at the temperature of the electric furnace or even at a lower temperature.

"This circumstance has not as yet been ascertained; in fact, one has rather good reason to maintain that in the voltaic are and in the electric furnace the tension of carbon

vapour is rather small.

"One may therefore hope, with sufficient grounds, that a simple rise in temperature may produce the fusion "

According to the above-mentioned deductions, Prof. La Rosa undertook some experimental researches with a view to explaining clearly his idea and consequently showing that the last deductions of Moissan are arbitrary.

By means of the musical arc + the author was able to

* "Sulla probabile fusione del carbonio nell' arco elettrico cantante e

nella scintilla." Nuovo Cimento, ser. 5, xviii. (1909). + "Trasformazione dell' arco elettrico cantante." Accademia dei Lincei, ser. 5, vii. (1908), oppure Ann. der Phys. iv. fol. xxix. p. 249 (1909).

Phil. Mag. S. 7. Vol. 7. No. 43. March 1929. $2 \mathrm{K}$ realize a higher temperature than that of the continuous are or of the electric furnace, because under the former conditions the amount of energy liberated within a certain time upon a unity of the gas mass, across which the discharge occurs, is greater than that which is consumed by the voltaic arc; consequently the intermittent arc gives thermic and luminous effects superior to those of the continuous arc. Furthermore, spectroscopic examination allowed the author to note that the spectrum of the intermittent arc is equal to that of an intense spark *. It was therefore natural that he should try to attain, by means of the musical arc, the liquid form of carbon.

Notwithstanding my wish to be concise, it is considered necessary, according to what will be said later, to recall the important experiment which permitted the realization of the

liquid phase of carbon.

An ordinary are with vertical carbons was employed. A tension of 300 volts was used in the main circuit, and a current variable from 1 to 20 amp. The branch circuit had a very small resistance and self-induction, and, furthermore, a capacity of 60 microfarads. The electrodes were immersed in an infusible crucible containing some pure sugar charcoal having a residue of 0.08 per cent. of ash, while the residues of the electrodes amounted to 0.33 per cent. After suitably regulating things, the arc was started. Above the two electrodes that remained unaltered were found incrustations of various forms and dimensions. Microscopic examination of these incrustations showed that they consisted mainly of a peripheral part, easily transformable into powder and made up of graphite particles showing the contour of the original charcoal particles, and of a central part lustrous, unctuous and very difficult to powder, and showing under the microscope no trace of the structure of the original charcoal particles. The ash residues left by the graphite thus obtained amounted to 0 05 per cent.

Researches of an experimental nature led the author to conclude that under the said conditions it is futile to think of a cementation of these various particles, due to their impurity, or to the phenomenon of real sublimation of carbon; moreover, repeating the experiment with the continuous arc under similar conditions, even for many hours, does not give the same results: graphite is no longer formed, but only the first example of incrustations above described.

Prof. La Rosa therefore arrives at the conclusion that

the particles of carbon attacked by the intermittent discharge

liquefy and weld themselves to the electrodes.

Later, by means of the Joule's effect*, the liquid form of carbon was attained by exploding a charcoal cylinder and passing across it brusquely a very intense current. Here, too, was confirmed by spectroscopic means the attainment of a greater degree of excitation †. This spectral method of excitation was later applied by Anderson to the study of spectra of other metals ‡.

Having thus obtained the liquid form of carbon, it was a question of making it crystallize in order to obtain the diamond; but the stable allotropic form of carbon at high temperatures is graphite, and under previous experimental conditions it was impossible to obtain crystallization, since after the arc was spent, the mass of charcoal remained, for a

certain time, incandescent.

It was necessary, consequently, to realize the conditions for fusion and to obtain at the same time a very rapid cooling.

Since the author had noted that the spectrum of the intermittent are is precisely like that of an intense spark, he substituted this for the arc, in order to obtain the desired

conditions.

The spark used in this experiment was charged with a battery of 72 large Leyden jars connected in parallel; the capacity of each was 500 cm. This battery was charged with a Klingelfuss coil of 80 cm. spark started by a medium current of 40 amp.

By this means he obtained particles of crystallized carbon of more or less spherical form, that of cut rubies. They were mainly of a greyish tinge; few were decidedly crystallized, transparent, of high refractive power, and of a density

greater than that of methyl iodide (CH₂I₂), (3.2).

A few of the most beautiful particles obtained are reproduced in the following microphotographs (Pl. VIII. figs. 1

& 2) kindly furnished by Prof. La Rosa.

The first of these constitutes a single crystal with curved facets, and is a form composed of two tetrahedra. The second looks like an aggregation of many minute crystals,

* "Sulla fusione del carbonio per mezzo dell' effetto Joule." Nuovo Cimento, ser. 5, xx. (October 1910).

† "Sullo spettro della luce che accompagna il riscaldamento elettrico di un bastoncino di carbone." Nuovo Cimento, ser. 5, xx. (Nov.-Dec. 1910).

† Anderson, "The Spectrum of Electrically-exploded Wires," Astroph. Journ. li. (1920). Anderson & Smith, "General Characteristics of Electrically-exploded Wires," Astroph. Journ. lxiv. (1926).

each of which forms two small pyramids united at their square base, or two tetrahedra. Accurate investigations completely exclude the possibility that these tiny crystals might be due to carborundum (CSi), the only composition that could have led us into error. The strongest proof was that of combustion. The above-described particles when burned in an atmosphere of oxygen left no residues of ash.

All these facts, therefore, allow us to conclude with sufficient assurance that the said particles are minute diamonds. These results were communicated by the author in 1909 to the Seventh International Congress of Applied Chemistry in London, and an account of these interesting experiments is published in 'Transactions of the Faraday Society,' vol. v.

pt. 2 (1910).

In a recent article that appeared in 'Nature,' May 19, 1928, was treated the problem of the artificial construction of diamonds. The author, who is known to us as C. H. D., refers to the experiments and results of various experimenters, among whom are Crookes, Parson, and others. He dwells mainly on the works of Mr. Parson and Mr. Duncan, who, as a conclusion to their researches in the field, believe themselves to be justified in affirming that no experimenter has as yet succeeded in producing diamonds in the laboratory, and that all who have heretofore prided themselves in having done so, have deceived themselves, since they have mistaken the crystals of spinel, whose properties are very similar to those of the diamond, for the diamond itself. Undoubtedly the works of Prof. La Rosa, to which reference has already been made, have escaped the attention of the above-mentioned authors as well as the author of the article in question.

While realizing the strength of the objections raised against the method of Moissan and his followers, it can be asserted

that these are groundless in the case examined.

In fact, with La Rosa's method we have not the insidious solution in fused metallic solvents as well as the possible formation of spinel crystals, because the particles that might contain traces of magnesia and aluminium are not present. Even when the presence of the crucible composed mainly of silicates is considered, the results obtained will not be weakened, since the crucible as well as the mass of carbon surrounding the attacked region brought about by the discharge remain unaltered at a temperature a little above that of the surrounding atmosphere; furthermore, the crystals obtained burn completely in an atmosphere of

oxygen, a result which I think is not obtained by any of the other methods.

These facts are strongly in favour of our thesis; hence the results of the experiments and the accuracy of the method are sure proofs which lead us to withdraw from the ultimate conclusion arrived at by (). H. D. It is believed, nevertheless, that the artificial construction of diamonds from the scientific point of view is no longer an unattainable goal, and that the difficulties that prevent the preparation of large and beautiful diamonds are only technical.

Reg. Università, Palermo. June 1928.

LVII. Thermodynamical Properties of the Electron, and Atomic Theory. By R. D. Kleeman, B.A., D.Sc.*

§ 1. Introductory Remarks.

IN a previous paper † the writer has made a beginning in the application of thermodynamics to the determination of some of the properties of the electron. In this paper the subject will be considerably extended, and an endeavour made to make it self contained, on account of its interest and importance.

§ 2. The Equation of State of an Electron Gas.

Let us consider a mass of electrons at any given density at T=0, or at the absolute zero of temperature. Since the electrons repel each other, a positive pressure is associated with the electronic substance under all conditions. Suppose that the substance is allowed to expand adiabatically. External work will be done during the process since the pressure is positive, and the temperature can therefore only decrease. But the temperature cannot fall below T=0, and hence does not change, which may be expressed in the important form that the various states corresponding to T=0 lie on an adiabatic.

Let us next integrate the well-known thermodynamical

equation

$$\left(\frac{\partial S}{\partial v}\right)_{T} = \left(\frac{\partial p}{\partial T}\right)_{v} \cdot \cdot \cdot \cdot \cdot \cdot (1)$$

^{*} Communicated by the Author. † Phil. Mag. vii. p. 53 (1929).

with respect to v between the limits ∞ and v, where S denotes the entropy of the electronic substance at the volume v, pressure p, and absolute temperature T, giving

$$S_{\infty} - S_{v} = \int_{v}^{\infty} \left(\frac{\partial p}{\partial T} \right)_{v} \cdot \partial v. \quad . \quad . \quad (2)$$

We may write the equation of state of the substance in the form

 $pv = RT \cdot \phi, \quad \dots \quad (3)$

where R is the gas constant, and ϕ a function of v and T. On substituting for p in the preceding equation, it becomes

$$S_{\infty} - S_{v} = \int_{v}^{\infty} \left(R\phi + RT \left(\frac{\partial \phi}{\partial T} \right)_{v} \right) \frac{\partial v}{v}. \qquad (4)$$

At T=0 we have seen that $S_{\infty} - S_{v} = 0$, and hence under this condition

$$\int_{v}^{\infty} \left(R\phi + RT \left(\frac{\partial \phi}{\partial T} \right)_{v} \right) \frac{\partial v}{v} = 0. \qquad (5)$$

If ϕ may be taken a constant when $v=\infty$, the value of the integral corresponding to the upper limit becomes infinite. The value of the integral corresponding to the lower limit evidently cannot be infinite for all values of v. Hence it follows that when $v=\infty$, ϕ is not a constant but a function of T, which becomes zero when T=0.

Next let us integrate the well-known thermodynamical equation

$$\left(\frac{\partial \mathbf{U}}{\partial v}\right)_{\mathbf{T}} = \mathbf{T} \left(\frac{\partial p}{\partial \mathbf{T}}\right)_{\mathbf{T}} - p \quad . \quad . \quad . \quad . \quad (6)$$

with respect to v between the limits ∞ and v, where U denotes internal energy of the substance, which gives an equation which may be written

$$U_{\infty} - U_{v} = RT^{2} \int_{v}^{\infty} \left(\frac{\partial \phi}{\partial T}\right)_{v} \cdot \frac{\partial v}{v}. \quad . \quad . \quad (7)$$

by means of equation (3). The left-hand side corresponds to the change in internal energy when the volume is increased from v to ∞ , and hence it cannot have an infinite value. Since ϕ is a function of T, it follows at once that when $v=\infty$, ϕ is a function of v as well as of T.

Now let us write

$$\phi = \frac{a}{v^a} + \frac{b}{v^\beta} + \dots \qquad (8)$$

in this equation, where a, b,... are functions of T and $\alpha, \beta,...$ constants, which gives

$$U_{\infty} - U_{v} = RT^{2} \left\{ -\frac{1}{\alpha} \frac{\partial \alpha}{\partial T} \frac{1}{v^{\alpha}} - \frac{1}{\beta} \frac{\partial b}{\partial T} \frac{1}{v^{\beta}} - \dots \right\}_{v}^{\infty}. \quad (9)$$

Since the integral corresponding to the upper limit cannot be infinite, it follows that α , β ,... must be positive quantities. Therefore, when the electrons are in the perfectly gaseous state (which corresponds to $v=\infty$), we may write

$$pv = RT\xi$$
, . . . (10)

where ξ decreases when v increases, and is less than unity.

§ 3. The Velocity of an Electron in an Electronic Gas decreases gradually between two Consecutive Collisions, und increases during a Collision.

Suppose that the velocity of an electron in an electron gas kept at constant volume remains constant between two consecutive collisions. On decreasing the volume of the gas this velocity is decreased, since, according to the previous section, the pressure decreases more rapidly than inversely as the volume.

The velocity thus depends on the length of the free path, which can be indicated, or measured, only by an internal change of the electron. Now suppose that an electron is allowed to pass into a vacuum. We may suppose that it is subjected along its path to a succession of electric fields, which alternately decrease and increase its velocity. Corresponding changes should occur in the electron which register these changes in velocity. It follows therefore that in general the velocity and internal conditions of an electron are related to each other; from which it follows that the velocity of an electron cannot remain constant between two consecutive collisions.

Next, let us suppose that the velocity of an electron increases between two consecutive collisions. The velocity will go on increasing unless reduced by each collision. Suppose that it is reduced to zero. On increasing the volume the pressure of the gas is increased under these conditions—which should not occur, and hence the velocity of an electron cannot increase between two consecutive collisions.

Lastly, suppose that the velocity is decreased between two consecutive collisions. The velocity must then increase during a collision, otherwise it will eventually become zero. On increasing the volume of the gas the average velocity decreases more rapidly than inversely as the volume, as should be the case.

Hence we have the important result that the velocity of an electron in an electron gas gradually decreases between two consecutive collisions, and increases during a collision.

This result will now be obtained in the next section in an

entirely different way.

§ 4. The Effect of the Radiation surrounding an Electron on its Kinetic Energy.

Electromagnetic radiatiation falling on a substance exerts a pressure upon it depending on the amount absorbed per second; thus, if p denotes the pressure per cm.² when the radiation is completely absorbed, it will exert the pressure 2p when it is completely reflected. An electron may thus be under the action of a number of forces, the resultant effect of which will now be investigated.

Suppose that some of the radiation surrounding the electrons of an electron gas, which is black-body radiation, is removed. The average velocity of the electrons will gradually decrease to correspond to the reduced density, or temperature, of the radiation. This will be brought about by the resultant radiation pressure on each electron

acting for a time contrary to its motion.

Next, suppose that some black-body radiation is added to the gas. The effect of the radiation will now for a time be

to increase the velocity of the electrons.

Now, the electrons in an electron gas possess velocities of all magnitudes, and a decrease or increase simply changes the distribution of the velocities. Therefore, if the radiation surrounding the electrons of an electron gas in equilibrium corresponding to a given temperature has no effect on the velocity of an electron, or if it is in equilibrium with the radiation, it will be in equilibrium with the radiation corresponding to a different temperature. Hence an increase or decrease in the temperature, or density, of the radiation would produce no effect on the velocity of each electron. But since an effect is produced, it indicates that each electron is subjected to a radiation pressure which continually changes its velocity. The direction in which this pressure, or the total force, acts would depend on the direction of motion of the electron; otherwise no force would be acting. It cannot be in the direction the electron is moving, because its velocity would then continually increase, reasoning along the same lines as in the previous section. Hence the force on the electron acts contrary to the direction of its motion, giving rise to a gradual decrease in its velocity. But since this cannot go on indefinitely, an increase in velocity must take place during each collision. Thus the result of the preceding section may also be obtained from radiation considerations.

It may be noted that equation (10) immediately follows, since under these conditions the pressure of the electron gas will decrease more rapidly than inversely as the volume.

§ 5. The Effect of the Radiation surrounding an Electron on its Internal Energy.

Suppose that an electric field is applied to an electron gas whose value is gradually increased to a maximum and then decreased to zero. The average velocity of the electrons is increased as a result, because more work is done on an electron when it moves in the direction the field is acting than when moving in the opposite direction, and hence the temperature will be raised. Hence an alternating electric field applied to an electron gas will gradually raise its

temperature.

These considerations apply also to the alternating fields due to the surrounding electromagnetic radiation. Since actually no rise in temperature takes place, it follows that we must look for other forces acting in addition on each electron. Now, the radiation falling on an electron when absorbed in part and stored up gives rise to a somewhat different force acting on it than the one considered. What may happen is shown by considering a plate of a substance placed in a radiation which is perfectly reflected by one side of the plate and absorbed in part by the other side. The energy absorbed by the plate per cm.2 if kept stationary is aE, where E denotes the energy per c.c. of the radiation, and α the fraction of the impinging radiation which is absorbed, and which is stored up in the plate in other forms of energy than kinetic energy of the plate as a whole. But radiant energy may be transformed into kinetic energy of the plate as a whole on allowing it to move under the radiation pressure. This pressure on the perfectly reflecting side is 2E and on the other side $E(2-\alpha)$, giving a difference in pressure equal to αE .

Now, in order that the electron may not continually increase in velocity due to the electric fields associated with the radiation, it must absorb radiant energy in the foregoing two ways. Hence the radiation surrounding an electron

continually changes its internal energy as well as its kinetic energy. Now, according to Section 4 the effect on the kinetic energy is to continually decrease it, which, however, is compensated by periodical increases taking place during collision. The internal energy will evidently continually increase. But since that cannot go on indefinitely, the internal energy must in part be periodically released. This can only happen during a collision. Hence we obtain the result that the internal energy of an electron in an electron gas increases between two consecutive collisions, and decreases during a collision.

§ 6. The Electric Field surrounding an Electron may undergo an Increase, during which it radiates some of its Internal Energy into Space.

We have seen in Sections 3 and 4 that the velocity of an electron gradually decreases between two consecutive collisions, and increases during a collision. This increase can only be brought about by a sudden increase in the electric fields of the colliding electrons. The increase in field must begin at the location of each electron and travel outwards with the velocity of light; in other words, a pulse of electric force would be propagated from each electron into space. It may be said to be caused by the electron passing from a state A to a state B. This change in state would be associated with a change in internal energy; evidently a decrease should take place which would appear as radiation. We have seen in the previous section that for other reasons a release of internal energy takes place during a collision.

Furthermore, since force and inertia will be involved in such a change of state, the electron is likely to vibrate about the state B, resulting in a succession of pulses being radiated

instead of a single pulse.

In the previous section we have seen that the internal energy is increased between two consecutive collisions, and more or less released during a collision. In this section it is shown how the accumulated internal energy is released, and in what form.

§ 7. An Electron may radiate in two ways.

It is well established theoretically and experimentally that

(a) An electron radiates when undergoing acceleration, and at the expense of its kinetic energy.

According to the previous section,

(b) An electron radiates when undergoing an internal change, and at the expense of its internal energy.

On both cases the radiation is likely to proceed according to the quantum equation

$$E_1 - E_2 = h\nu$$
, (11)

where E_1-E_2 denotes the change in energy, ν the frequency, and h Planck's constant. This equation is known to hold under certain conditions. The energy is radiated as a continuous train of waves, for if it were radiated as two trains with an interval between, each would be radiated according to this equation and hence with different frequencies*.

Since the absorption of internal energy by an electron in an electron gas takes place over its mean free path, it can only be gradual, or the absorption of radiation by an electron takes place in indefinite amounts. When the kinetic energy of an electron in an electron gas is increased by the surrounding radiation on increasing its density, this increase may evidently take place in indefinite amounts.

§ 8. The Equilibrium between the Emission and Absorption by the Electrons in an Electron Gas.

When an electron undergoes an acceleration a for the time ∂t it radiates the energy $\frac{2}{3}\frac{e^2a^2}{e}$. ∂t into space, according to the process (a) in the preceding section, where c denotes the velocity of light and e the electronic charge. It also radiates the energy I. $\frac{\partial u}{\partial t}$. ∂t , according to the process (b), where u denotes its internal energy and I is an appropriate factor. The energy for these radiations is derived from the surrounding radiation, which exerts a force f on the electron giving rise to an absorption of energy equal to f. $\frac{\partial x}{\partial t}$. ∂t , where ∂x denotes the distance traversed during the time ∂t . An amount of energy X. ∂t is absorbed which does not affect the velocity of the electron but appears as internal energy, where X is an appropriate factor of ∂t . Lastly, the

^{*} In a subsequent paper it will be shown that a part of the kinetic energy and a part of the internal energy may be radiated as a whole like a part of the internal energy.

external electric field gives rise to a force F acting on the electron which performs work upon it equal to F. $\frac{\partial x}{\partial t}$. $\frac{\partial x}{\partial t}$.

$$\frac{2}{3}\frac{ea^2}{c} + I \cdot \frac{\partial u}{\partial t} = f \cdot \frac{\partial x}{\partial t} + F \cdot \frac{\partial x}{\partial t} + X. \quad . \quad (12)$$

Since

$$a = \frac{\mathbf{F} + f}{m}$$

and

$$V = \frac{\partial x}{\partial t}$$
,

where V denotes the velocity of the electron and m its mass, equation (12) may be written in the general form

$$\psi(F, f, u, V, I, X) = 0.$$
 . (13)

It can be shown that I and X are functions of F, f, u, and V. The internal energy of an electron cannot increase indefinitely, and hence the change in internal energy $\frac{\partial u}{\partial x}$ will be a function of the internal energy. When the velocity is zero u cannot continually decrease or increase, or u does not change independently under these conditions, and hence $\frac{\partial u}{\partial x}$

is also a function of V. We have seen that when two electrons get sufficiently close to each other the change in internal energy that takes place is attended by a change in external field. Hence we also have that the change is a function of F as well as of f. Finally, we may therefore write

$$\frac{\partial u}{\partial x} = \psi_1(\mathbf{F}, f, \mathbf{V}, u), \dots (14)$$

from which it follows that in general

$$\psi_2(\mathbf{F}, f, \mathbf{V}, u) = 0.$$
 (15)

A comparison of this equation with equation (13) shows that I and X are functions of F, f, u, and V.

Equation (15) expresses the important result that f, F, V, and u are related to each other. An important consequence is that potential energy of attraction may be converted into internal energy. Thus, suppose that an electron initially at rest and at infinite distance from a positive charge moves towards it and becomes stationary at a certain distance,

being now not under the action of any force. The work done on the electron by the electric field appears as internal energy of the electron and the positive charge. If this energy be now radiated into space the two charges will again exert a force upon each other according to the well-known laws. Something similar no doubt happens during the collision of two electrons, which we have seen is attended by an increase in their electric fields. This probably means that the field of an electron gradually decreases between two consecutive collisions, and then increases to its previous value during collision. If that were not so, two electrons in an electron gas would never get near to each other when their velocity is small.

Further consequences of the relation will be discussed in the next sections. It should be possible to devise laboratory experiments to test directly the existence of such a relation.

§ 9. 1 he Formation of Atoms.

Let us consider what may happen to an electron moving more or less in the direction of a stationary positive charge. The effect of the surrounding radiation, according to Sections 3, 4, and 5, is to continually increase the internal energy of the electron and to decrease its velocity. The effect of the field due to the positive charge will, however, be to increase the velocity. The path described will be of the nature of a spiral. Now, it may happen, according to Sections 3 and 4, that the velocity of the electron after a time becomes zero; it will then, of course, not be under the action of an electric force, which is possible according to equation (15). But it will be under the action of a force when displaced from its position of equilibrium.

Suppose, now, that a second electron is brought near this electrical doublet. The electron of the doublet will now be under the action of a force, because the position it occupies is not the one to which it would have settled down when under the simultaneous action of the positive charge and the

extraneous electron in its position.

If the extraneous electron is brought sufficiently near to the doublet, the electron of the doublet might get displaced into a new position of equilibrium at which it would not be under the action of a force, being the position to which it would have settled down under the influence of the positive charge and the extraneous electron.

If the second position of the electron of the doublet corresponds to a smaller value of its internal energy than it possessed in its previous position, the difference during the displacement is radiated into space according to the process

(b) in Section 7.

The foregoing considerations may be applied to several electrons initially in motion and gradually settling down to stationary positions under the action of a positive charge. The atoms might have been formed in this way, the system gradually changing from a dynamic to a static state *. If two atoms, or an atom and an electron, are brought close to one another, the electrons will be put under constraint, which may result in new positions being occupied and radiations emitted during the process. When the external influences on an atom are removed the displaced electrons which radiated internal energy gradually regain their internal energy by absorption from the surrounding radiation (Section 5), and gradually slip back to their previous positions during the process. The other displaced electrons are likely to slip back to their positions as soon as the external force is removed, without the displacement in either direction being attended by radiation.

An electron in an atom may gradually absorb energy from the surrounding radiation and gradually occupy a new position of equilibrium. But this position is likely to be unstable, and the electron might on account of some disturbance or other easily slip back to its previous position. But this process would be rapid, since the difference in internal energy between the two positions would be radiated

as a continuous train of waves of energy $h\nu$.

§ 10. Application to the Bohr, Lewis, and Langmuir Atom.

The previous investigations by the above-mentioned scientists on the spectral and chemical relations of the atom in connexion with their electronic configuration throw some further light on the process of atomic formation as outlined in the previous section. Thus, consider an electron describing a Bohr circular orbit around a positive charge. The electron will gradually slow down and reach a stationary position, according to Sections 3 and 4. In order that its orbit may remain unchanged the electric force F acting on it due to the positive charge will, according to equation (15), gradually decrease to zero when the electron is stationary, and accordingly be given by

$$\mathbf{F} = \frac{m\mathbf{V}^2}{r}, \quad . \quad . \quad . \quad . \quad (16)$$

^{*} In a subsequent paper another method of formation will be pointed out.

where r denotes the radius of the orbit of the electron. The internal energy of the electron will increase in value equal to the initial kinetic energy, since in the end this is equal to zero, and hence we may write

$$\partial u = \partial (\frac{1}{2}m\nabla^2) = m\nabla \cdot \partial V.$$
 (17)

The kinetic energy is actually lost by radiation into space, and in doing work against the force f acting contrary to the motion of the electron, which slows it down, according to Section 4. We accordingly have

$$\frac{2}{3} \frac{e^2 a^2}{c} \cdot \partial t + f \nabla \cdot \partial t = \frac{\partial}{\partial t} (\frac{1}{2} m \nabla^2) \partial t$$

$$\frac{2}{3} \frac{e^2 \nabla^3}{cr^2} + f = m \frac{\partial \nabla}{\partial t}, \qquad (18)$$

$$a = \frac{\nabla^2}{a}.$$

since

or

The energy, is, however, recovered again through absorption of energy from the surrounding radiation, according to equation (12). In this manner the non-radiating Bohr orbit

is explained.

When an electron is transferred from one Bohr orbit to another, the difference in internal energy is radiated into space by the process (b) in Section 7. This difference in energy, according to Bohr, consists of the difference between the energies required to move the electron to rest at infinity. From this it follows that the potential energy of an electron becomes internal energy, when through a change in internal energy the electric force acting upon it decreases. This possibility has already been indicated in Section 8. Thus the process of radiation of a Bohr atom is explained.

Similar considerations may be applied to the Sommerfeld elliptical orbits of an electron, and to atoms involving more than one electronic orbit. We have seen that the dynamic atom of Bohr ultimately becomes a static atom; and this may now fulfil all the conditions demanded by the Lewis-Langmuir theory of the static atom in connexion with its various chemical properties. The two rival theories may

now be said to be completely reconciled.

It appears from this paper that electromagnetic radiation may spread evenly in space, and may be absorbed by electrons and atoms in indefinite amounts, which appears in the form of internal energy. This energy may be radiated in part or altogether into space, but only as a continuous train of waves whose energy is equal to $h\nu$. With this as basis all radiation phenomena may be explained, of which an outline has been given in this paper. In a subsequent paper this will be further carried out in connexion with radioactivity, ionization, the Compton effect, etc.

Schenectady, N.Y., U.S.A.

LVIII. On a Supposed Limitation of the Second Law of Thermodynamics. By Charles Coolman, Lecturer in Physics in the Agricultural Timiriasev Academy of Moscow*.

Cairbournet tried to show theoretically that some considerations of molecular kinetic character lead to the possibility of establishing a transgression of the second Law of Thermodynamics. According to his statement, a funnel-like aperture allows to pass in one direction more molecules than in the other, under pressure, when the mean length of the free path is large as compared with the dimensions of the funnel; this produces a spontaneous

difference of pressure at both sides of the funnel.

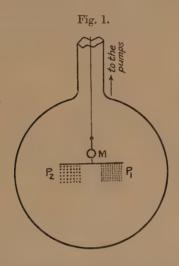
R. d'E. Atkinson ‡, Witmer §, and the author ||, the last in a more general way, have proved the error of Fairbourne's conclusions. As I have shown (l.c.), this problem, owing to the insufficiency of our knowledge of the laws of reflexion of the molecules from a real wall, cannot be solved from a molecular-kinetic standpoint, in spite of J. W. Fisher's ¶ opinion. Gaede ***, confirming the existence of a minimum conductivity of gases (in the function of pressure), assumed as its only explanation the directing influence of the surface film. He calculated the resulting reciprocal repulsion of two plates (at the same temperature), and showed on a lever-balance (sensibility not indicated), that the effect was, at any rate, a thousand times smaller than

^{*} Communicated by the Author.
† Phil. Mag. (6) xliii. p. 1047 (1922).
† 'Nature,' iii. p. 326 (1923).
§ Phil. Mag. (6) xlvii. p. 152 (1924).
| Is sent to the J. Russ. Ph.-Chem. Soc.
¶ Phil. Mag. (6) xlvii. p. 152 (1924).
** Annalen d. Ph. xli. p. 289 (1913).

the calculated one. From this observation of Gaede we may conclude that a funnel-like aperture might cause a motion of

the gas in one preponderating direction.

On the other hand, J. Kossonogov * published an article where he describes a similar phenomenon observed by him in 1920, before Fairbourne's publications. My own careful investigations (l.c.) did not confirm the results obtained by him. As the experiments of Kossonogov were conducted under atmospheric pressure and Gaede's method was not sufficiently sensitive, it was interesting to make the following controlling experiments.

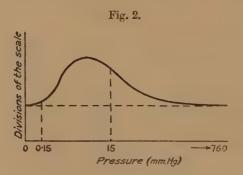


In the centre of a large bolt-head (diameter about 20 c.) a system was suspended on a quartz thread consisting of a little rod with a mirror M, at the ends of which were attached two thin porous glass plates, P_1 and P_2 (2 c. \times 2 c. each), with conical apertures (fig. 1). The porous plates (made by Schott, Jena, 1925) were attached in such a manner that the narrower ends of the conical pores of plate P_1 were turned in the opposite direction to the narrower ends of the pores of the other plate, P_2 .

If through each of these conical apertures a greater quantity of molecules flowed in one direction than in the opposite, an easily discoverable rotational moment would be imparted to the system. The diameter of the narrower ends of the pores

^{*} J. Russ. Ph.-Chem. Soc. lvi. pp. 25-39 (1924).

was $4-5\,\mu$ (the mean free path of the molecules of air at 15-18 mm. Hg was the same). The diameter of the larger pores was about $100\,\mu$ (corresponding to the pressure of 0.15 mm. Hg). The maximum of declination had to be looked for in these limits. A position when the system was not declined could exist at a very strong rarefaction (according to Gaede the directing action of the surface film disappeared already at 0.001 mm. Hg), or, with some approximation, at the atmospheric pressure. In that case, according to Fairbourne and Kossonogov, the curve would be represented by fig. 2. The period of vibration was about 4 minutes. The moment of inertia was reckoned at 9 C.G.S units. With a scale placed at a distance of 2 m., one division of the scale corresponded to 1.4.10-6 dyn. cm.



This, at the maximum of the effect sought for, corresponds for molecules reflected in one direction to the prevalence of one hundred millionth of all molecules fallen. Because of the great sensibility of the method, a good thermal isolation proved to be necessary. It was secured by enveloping the bolt-head in several layers of cotton with interlayers of tinfoil; in front of the window a water cuvette was placed and a scale of total internal reflexion was used.

Three series of observations with a good constancy of the zero in limits from 10⁻⁷ mm. to 760 mm. Hg showed a complete absence of the effect. An artificial change of temperature in the room showed that the natural variations of temperature could not conceal the effect if such existed.

The same results were obtained with little diaphragmlike wings made of porous plates composed of two layers of gypsum and cement, like those used by Kossonogov. Evidently the directing action of the surface film inferred by Gaede (if it exists) in this case cannot lead to a limitation of

the second law because of the unevenness of the surface and perhaps of the oscillations of the surface film itself.

Thus the cases of limitation of the second Law of Thermodynamics pointed out by the above-mentioned authors must be considered theoretically and experimentally as inconsistent.

I take this opportunity for expressing here my profound regret at the recent death of Wladimir Alexandrovitch Michelson (Feb. 27th. 1927), Professor of Physics and head of the Physical Laboratory, who placed at my disposal all facilities for carrying out this work.

Physical Laboratory. Agricultural Timiriasev Academy, Moscow, August 1927.

LIX. Precision Measurements of X-Ray Reflexions from Crystal Powders. The Lattice Constants of Zinc Carbonate, Manganese Carbonate, and Cadmium Oxide. By J. BRENTANO, D.Sc., Lecturer in Physics, Manchester University, and J. ADAMSON, M.Sc., Research Student. Manchester University*.

SUMMARY.

EXACT determinations of the spacing and of the rhombohe Iral angle of zinc carbonate and of manganese carbonate, derived from X-ray measurements on crystalline powders, are added to analogous determinations for magnesium carbonate given in a previous paper.

The results are used to verify calculated lattice constants of the carbonate series. Introducing our values, it is found that calculations in which a fixed parameter for the CO3 group is assumed give good approximation but do not quite agree with the observed data. This indicates that terms characteristic of the various elements have to be considered.

Determinations are made showing the accuracy of the method used. With this method, satisfying a focussing condition, uniformly spaced lines of constant width are obtained and a standardizing substance of great volume absorption is introduced. Some points arising in connexion with its application are discussed.

^{*} Communicated by the Authors.

A determination of the lattice constant of cadmium oxide is made in order to use cadmium oxide as standardizing substance.

- 1. IN a recent paper by one of us and by W. E. Dawson *, exact measurements of the lattice constants of magnesium carbonate were discussed. These measurements introduced an appreciable change in the spacing, bringing magnesium carbonate and calcite, the two extreme members of the isomorphous series of carbonates, into close agreement with respect to calculated values + of their lattice constants derived on the assumptions developed by W. L. Bragg and S. Chapman. The discrepancies of intermediate members are of the same order as the correction introduced in the magnesium carbonate spacing, and it seemed of interest to examine to which extent they might be due to the uncertainty of the lattice measurements. The new determinations were made with a microcrystalline powder, using an X-ray method, and the rhombohedral angle was derived from a consideration of those lattice spacings, the ratios of which were most sensitive to changes in the angle.
- 2. In the case of a trigonal system, the side α of the elementary cleavage rhomb and the rhombohedral angle α can be calculated by introducing in the relation,

$$d_{hkl} = \frac{\alpha \sqrt{1 + 2\cos^3 \alpha - 3\cos^2 \alpha}}{\sqrt{(h^2 + k^2 + l^2)\sin^2 \alpha + 2(hk + kl + hl)(\cos^2 \alpha - \cos \alpha)}},$$
. . . (1)

the values of the lattice spacings d_{MM} which have been

obtained from X-ray reflexions $(h\vec{kl})$.

Referring more particularly to the determination of the angle α , this can be derived from the ratio of any two spacings, and spacings may be called sensitive for this purpose when they can be combined so that their ratio varies greatly with variation of the angle α . In the case of the isomorphous carbonates of the calcite type, the angle α is about 103°. In these circumstances, as will be seen from equation (1), an appreciable variation in α is only accompanied by a relatively small change in the corresponding values of d_{hkl} . For example, in the case of magnesium

^{*} J. Brentano & W. E. Dawson, Phil. Mag. (7) iii, p. 411 (1927).
† S. Chapman, J. Topping, and J. Morral, R. Soc. Proc. A, cvi. p. 369 (1924).

carbonate, a change of 0° ·l in the angle α corresponds to a change of 0.0073 in the relative magnitude of the most sensitive spacings. In order therefore to determine the rhombohedral angle, a high degree of accuracy is required

in the measurement of the lattice spacings.

We have now measured the lattice constants of zinc carbonate and of manganese carbonate, in order to obtain more exact data for these substances. Since specimens of manganese carbonate are available which permit of exact goniometric measurement, in this case only the value of a was required. We have, however, chosen to determine the rhombohedral angle as well, as a test of the method, and have compared this value derived from the X-ray data with the results obtained by goniometric measurements. Both sets of measurements were carried out on the same crystal specimen.

The method used is based on the general focussing condition for the reflexion from a powder, to which attention has been drawn in an early paper by one of us *, which in

the case of a flat layer leads to

$$\frac{\sin\alpha}{\sin\beta} = \frac{a}{b}.$$

 α and β are here the angles which the incident and emergent beams respectively make with the surface of the layer, α and b are the distances of the layer from the entrance slit defining the source of the X-ray beam and from the surface recording the reflexions.

The general arrangement of the instrument has been

described in a previous paper.

The advantages of this method for making exact measurements depend upon the fact that the reflexions are recorded in such a way as to establish linear relations between the angles of deflexion and the corresponding distances of the lines measured on the film, permitting of direct interpolation between the distances and the angles. Further, when satisfying the focusing condition, symmetrically shaped lines of practically constant width are obtained. These two factors are of great assistance in making absolute measurements of the actual angles of reflexion or in determining the relative position of the lines with respect to a known reference pattern. It must be noted, however, that full symmetry of the lines is only obtained with a thin powder layer which also enables exposures to be taken, using in turn

^{*} J. Brentano, Arch. Sc. Phys. et Nat. (5) i. p. 550 (1919); Proc Phys. Soc. London, xxxvii. p. 184 (1925).

both of its sides, thus eliminating any error due to imperfect centring. In the present case it was decided not to undertake absolute measurements, but to adopt the method which had been used in determining the lattice constants of magnesium carbonate. A standardizing substance was mixed with the powder of the specimen, and a calibration curve prepared in which the observed positions of the standardizing lines are plotted against the angles calculated from its known spacing.

The calibration curve is almost a straight line, and the angle of deflexion associated with any point of the film can be obtained with great accuracy. Any systematic errors of the instrument are thus eliminated, and thick layers can be used as long as the absorption in the layer is sufficiently great and the camera sufficiently large to make the asymmetry due to penetration of the X-rays in the layer

negligible.

3. The structure of zinc carbonate has been determined by W. L. Bragg *, using an ionization spectrometer and a single crystal, and by R. W. G. Wyckoff †, using the Laue method. G. R. Levi and A. Ferrari ! have also made a determination of the edge of the elementary cleavage rhomb, using a crystal powder and the Debye method. In no case was a determination of the rhombohedral angle made.

Our evaluation of the rhombohedral angle a respectively of the angle between adjacent faces of the rhomb ϵ was carried out, using the diagram reproduced in the previous paper §, in which quantities proportional to the logarithms

	TABLE I.	
L.		Indices.
9.1930	***************	311
9.2732	***************	$22\overline{2}, \ 2\overline{2}\overline{2}$
9.3080	***************************************	311
9.4238	***************************************	313
9.4725	***************	$40\overline{2}$
9.4948	***************************************	400
9.5152	****************	$4\mathbf{\bar{2}}\mathbf{\bar{2}}$
9.6245	*****************	333, 333
9.7124		511

^{*} W. L. Bragg, Proc. R. Soc. A, lxxxix. p. 468 (1914). † R. W. G. Wyckoff, Amer. Jour. Sc. 1. p. 317 (1920). † Levi & Ferrari, N. Cim. xxxiii. p. 516 (1924).

Brentano & Dawson, l. c.

of the spacings are introduced, so that differences are substituted for their ratios.

In Table I. we give our experimental values for the glancing angles θ in the form L=2 log sin θ , this being

the expression plotted in the diagram.

The value thus obtained for the rhombohedral angle is 103°27'; it seems therefore unnecessary to suggest any alteration in the usually accepted value 103°28' based on a goniometric determination by G. Rose*.

The value for the d_{100} spacing is

$$d_{100} = 5.493 \pm 0.005 \,\text{Å}.$$

as compared with the value $d_{100} = 5.44 \,\text{Å}$. obtained by Levi and Ferrari.

The value for the side of the elementary cleavage rhomb becomes

 $a = 5.928 \pm 0.005 \,\text{Å.},$

and the volume of the elementary cleavage rhomb

$$v = 1.877 \pm 0.015.10^{-22}$$
 cm.³,

giving a density of the pure zinc carbonate of 4.406 g./cm.³ This value is lower than the value 4.51 obtained by Levi and Ferrari from their lower value of the d_{100} spacing. Direct density measurements on specimens of zinc carbonate range from 4.30 to 4.45.

4. The structure of manganese carbonate has been determined by W. L. Bragg †, using the ionization spectrometer, and by R. W. G. Wyckoff ‡, using the Laue method.

The results of our measurements are given in Table II.

TABLE II.									
L.		Indices.							
9.4003		221							
9.4475	•••••	3 21							
9.4956	********	$42\overline{2}$							
9.5413	**************	2 22							
9.5813	•••••	$33\overline{1}$							
9 6017	••••	333, 3 3 3							
9.6616	***************************************	500							
9.6960		$42\overline{4}$							

^{*} G. Rose, Pogg. Ann. lxxxv. p. 132 (1852).

[†] L. c. ‡ L. c.

The value obtained for the rhombohedral angle is

$$\alpha = 102^{\circ} 50'$$
.

An accurate goniometric determination of the angle a, for which we are indebted to Mr. H. E. Buckley, M.Sc., made on the specimen from which the powder was prepared, gave the value 102° 50.3', which agrees with our determination within one minute of arc. These values also agree with the usually accepted value of $\alpha = 102^{\circ}50'$ obtained by A. de Schulten *.

For the value of the d_{100} spacing we obtain

$$d_{100} = 5.666 \pm 0.005 \,\text{Å}.$$

The value for the side of the elementary cleavage rhomb becomes a = 6.064 + 0.005 Å.

compared with the value $a = 6.04 \,\text{Å}$. obtained by Wyckoff, and the volume of the elementary cleavage rhomb

$$v = 2.039 \pm 0.015 \cdot 10^{-22} \,\mathrm{cm.}^3$$

leading to a density of 3.747 g./cm.3 The values of the direct density determinations on crystal specimens range from 3.47 to 3.76. The variations are mainly due to contamination by other members of the isomorphous series.

5. The determinations of the rhombohedral angles confirm in each case the generally accepted values also for the carbonates of magnesium and of zinc, for which the goniometric determination is more difficult, owing to the imperfections of the crystal and any effect of contamination more likely to occur. It is found more particularly that these values are actually the values of the rhombohedral angles for the crystal specimens for which the determinations of the lattice spacings were carried out. On the other hand, our measurements of the spacing d_{100} , respectively of the side a of the elementary rhombohedron lead to values which in each case differ appreciably from the values obtained in earlier determinations.

In papers by W. L. Bragg and S. Chapman + and by S. Chapman, J. Morral, and J. Topping ‡, the rhombohedral angles of the series of isomorphous carbonates, which includes the carbonates of magnesium, of zinc, and of

^{*} A. de Schulten, Bull. Soc. f. Min. xx. p. 196 (1897). † W. L. Bragg and S. Chapman, Proc. R. Soc. A, evi. p. 369 (1924). † S. Chapman, J. Topping, and J. Morrall, Proc. R. Soc. A, cxi. p. 25 (1926).

manganese, are derived by considering the condition of minimum potential energy of the intermolecular electrostatic fields. In these calculations the virtual distance between the C and the O atoms is introduced as a constant parameter for the whole series of carbonates. Using earlier values for the spacing a, the calculated rhombohedral angles show the observed decrease of the angle in passing from the lighter to the heavier carbonates, but do not arrive at a full

numerical agreement with the experimental values.

In the earlier paper discussing the accurate values for the lattice constants of magnesium carbonate and giving prominence to the two extreme members of the series, magnesite and calcite, for which exact lattice measurements were available, we have pointed out that very close agreement between the calculated and the observed values could be obtained by a slight reduction of the virtual distance of the C and O atoms. The then resulting discrepancies for other members could possibly be attributed to the uncertainty of the lattice constants.

Introducing our new values for the spacing of zinc carbonate and of manganese carbonate in the calculation, we can compare the resulting rhombohedral angles with the

experimental values.

The results are given in Table III. The data previously obtained for magnesium carbonate and those for calcite are added; for the latter obviously no question of accuracy in the determination of the spacing arises.

TABLE III.

	a.	a'.	a,	a.	a'.	æ,	₩,
Magnesium carbonate	5.86	4.61	$10\overset{\circ}{3}2\overset{\circ}{8}$	5.899	4.627	$10\overset{\circ}{3}\ 2\overset{\prime}{4}$	103 19
Zinc carbonate	5.90	4.64	103 18	5.928	4.654	103 16	103 28
Manganese carbonate	6.10	4.77	$102\ 52$	6.064	4.740	102 58	102 50
Calcite	6.38	4.96	102 4		_		101 55

In the first and second columns the side of the elementary cleavage rhomb a and the distance between adjacent metal atoms a' are given as assumed by Chapman, Topping, and Morral; in the third the values of the rhombohedral angle α so calculated. In the fourth, fifth, and sixth columns are given the corresponding values obtained from our lattice measurements. In the last column are plotted the observed rhombohedral angles.

It will be seen that by using our values for the spacing a', the calculation of the rhombohedral angles leads to values

which for magnesium and manganese carbonate are in each case slightly greater than the values actually observed, the difference being 5' and 8' respectively. For zinc carbonate, on the other hand, the calculated value is smaller than the

observed value, the difference being 12'.

Of the two carbonates for which exact measurements have now been made, manganese carbonate falls thus in line with magnesite and calcite, the difference between the calculated and the observed rhombohedral angles being of the same order and in the same direction in each case, the calculated angle being larger. By a slight reduction of the constant parameter the residual differences could be brought within

the limits of the possible experimental errors.

The resulting discrepancy for zinc carbonate would then be increased. This discrepancy is too great to be accounted for by experimental errors. The assumptions on which the calculation is based are thus able to give a remarkably good approximation, but are not sufficient to determine fully the configuration of the crystals of the carbonate series. It will also be seen that an agreement between the calculated and the observed values could not be obtained by introducing a parameter for the effective size of the CO₃ group decreasing regularly with greater atomic weight of the metal. On the basis of the assumptions made in the theory, it would appear that terms for the field distribution characteristic to each metal have to be introduced.

6. The determinations were made on powders, partly owing to the imperfections of the crystals available in the case of magnesium carbonate and of zinc carbonate, partly with the object of using a great number of reflexions for the measurement.

The accuracy of the method used has been discussed in the previous paper, by referring to the agreement between the measurements of the individual lines of magnesium carbonate. It is now indicated in a different way by the agreement of the X-ray determination of the rhombohedral angle of manganese with the result of the direct goniometric measurement, the two values being 102° 50′ and 102° 50·3′ respectively.

A few points may be discussed which have to be considered

when making exact measurements from powders.

As has been mentioned above, two essential conditions for an exact evaluation of the X-ray reflexions are to obtain photographic records permitting of a linear interpolation between the distances of the lines measured on the film and the angles of reflexion, and to avoid an asymmetry of the lines due to absorption of the X-rays in the deeper parts of the layer, which introduces errors when strong and weak lines have to be compared. The first condition is satisfied in the particular method used for our measurements. The second condition, to avoid too great a penetration of the X-rays into the crystal powder, can be attained by admixing a standardizing substance of great volume absorption and great volume scattering. By this means the thickness of the layer from which the X-rays are scattered to any appreciable extent can be reduced, and if the standardizing substance presents strong volume scattering, only a small fraction of the reduced volume of the layer, determined by the requirements of sufficient absorption, needs to be occupied by the standardizing substance, which helps to shorten the time of exposure when examining powders constituted of light elements. The reference substance should give a simple pattern, and should be constituted of such elements that their characteristic radiation is not excited by the X-ray radiation used.

7. We have found that nickel oxide in conjunction with Cu $K\alpha$ radiation satisfies the requirements of a standardizing substance, and this was the reason for making a more exact determination of its lattice constant.

In connexion with other work which is in progress, we were led to search for an alternative substance applicable to a range of shorter wave-lengths which would excite the characteristic radiation of nickel. Of a number of substances which we examined for this purpose, cadmium oxide was found most suitable for satisfying the required conditions."

Cadmium oxide can be used for the K radiation of copper, molybdenum, rhodium, and silver; its structure, like the structure of nickel oxide, is of rock-salt type; it gives thus the same pattern of lines, but, owing to the larger spacing, the cadmium-oxide reflexions occur at somewhat smaller angles than the corresponding reflexions of nickel oxide.

A determination of the cadmium oxide spacing by the method just described, with rock-salt as reference substance, gave

 $d_{100} = 4.683 + 0.004 \text{ Å}.$

Our value is appreciably smaller than the value 4.72 obtained by Davey and Hoffman*, and obtained again by

^{*} W. P. Davey and E. C. Hoffmann, Phys. Rev. xv. p. 333 (1920).

P. Scherrer*. A smaller value, 4.699, had been observed

by H. P. Walmsley †.

The cadmium-oxide powder examined by us showed that a certain part of it consisted of very small particles, so that the lines for the larger angles of deflexion were somewhat blurred. This blurring impairs exact measurements, but only where very great accuracy is required; the two a lines of Cu K radiation, for instance, were quite well separated.

8. In such cases where an exact measurement of a lattice constant is required, and not the determination of a structure, this can often be based on comparative measurements of a group of lines situated within a small angular range in the region of large angles of deflexion. The photographic film can then be placed at a considerable distance from the powder layer, so that the penetration of the rays into the layer has very little effect. In this case it is of advantage to make the distance b of the powder from the photographic film greater than the distance a of the entrance slit from the powder and to satisfy the focusing condition for the ratio a/b chosen. In fact, for a given aperture of the beams, other conditions being the same, the expression for the intensity of the reflected radiation a contains a factor

$$1 + \frac{\sin \alpha}{\sin \overline{\beta}},$$

which becomes greater when $\alpha < \beta$ or the distance a < b; this makes it possible to increase the distance between the photographic film and the powder layer without arriving to

unduly long exposures.

From the focussing condition it will be seen that with a given orientation of the powder layer, i.e. a given angle of incidence α , the angle β , for which the definition is sharp, varies with b. When satisfying the focussing condition for a film situated at a great distance, for which β and therefore the total angle of deflexion is large, the focussing condition is simultaneously satisfied for a recording surface situated nearer to the powder with a smaller β and thus a smaller angle of deflexion. By disposing two concentric films at different distances, it is thus possible to obtain sharp definition for two different angular ranges at the same time.

^{*} P. Scherrer, Zeitschr. f. Kryst. lvii. p. 196 (1922).
† H. P. Walmsley, Proc. Phys. Soc. xl. (1927).
† J. Brentano, Phil. Mag. vi. p. 183 (1928).

When determining the lattice constant of cadmium oxide, the distance a of the entrance slit from the powder was 13.5 cm., the distance b of the main film from the powder was 17.0 cm., and an auxiliary film was interposed at a distance of 5.3 cm. from the axis of rotation of the powder; this film was not protected by a rotating screen, and the lines were more or less blurred outside the region of correct definition.

Only the outer film was used for the actual determinations, but the auxiliary film was convenient for checking the identification of the lines.

In conclusion, we wish to express our thanks to Professor W. L. Bragg, F.R.S., for various facilities, including the use of a transformer obtained by means of a grant of the Royal Society.

Manchester, Dec. 15th, 1928.

LX. The Distribution of Electrons in Atoms. By LOWELL M. ALEXANDER, Ph.D., Associate Professor of Physics, University of Cincinnati*.

T.

STONER+ makes the assumption that the number of electrons in an atom associated with each sub-level is equal to double the inner quantum number. This gives for the first three levels:—K, 2; L_{II}, 2; L_{III}, 4; M_I, 2; M_{II}, 2; M_{III}, 4; M_{IV}, 4; M_V, 6. Stoner supports this distribution of electrons by evidence of an indirect nature, e. g., the intensity of X-ray lines, absorption of X-rays, magnetic properties, chemical properties, optical spectra, etc.

TT.

In Bohr's picture of the atom, the relation between sublevels of a group should be similar to the relation between the principal groups. As the K and L groups approach

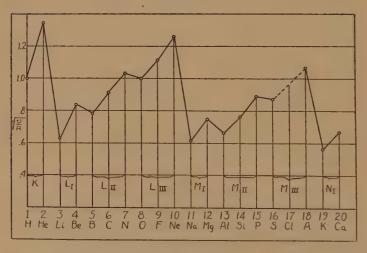
^{*} Communicated by the Author.

[†] Stoner, Phil. Mag. xlviii. p. 719 (1924).

completion, the energy necessary to remove the most lightly bound electron shows an upward trend, reaching maxima in helium and neon, the elements which complete the K and L groups respectively. The outer electron in the next elements (lithium and sodium) is the first electron of the next principal group, and since it is bound on the atom in an orbit of large radius, the energy necessary to remove it is small—in fact, a minimum.

We should expect this same phenomenon to occur on a smaller scale as the sub-levels of a particular group are formed. For example, in the formation of the L group one

Fig. 1.



would expect the ionization potential to increase for successive atoms until the L group is complete. The outer electron in the atom of next atomic number is the first electron of the L_{II} sub-level and would be bound on the atom in an orbit of greater radius than the L_I electrons. Thus at this point we might expect a decrease in the ionization potential, although the magnitude of this decrease would be considerably smaller than the corresponding decrease between principal groups. It seems possible in this way to get direct evidence as to the distribution of electrons in sub-levels. The broken line of

fig. 1 represents the square root of the energy
$$\left(\sqrt{\frac{\nu}{R}}\right)$$

necessary to remove the most lightly bound electron for the elements from H (1) to Ca (20). For brevity, I shall denote $\mathbf{X_1}$ as the energy necessary to remove the most lightly bound electron in the element X. It is seen from the figure that He₁ is greater than H₁, indicating that the second electron in helium is associated with the first to form the K group. Li₁ is considerably smaller than He₁, indicating that the third electron of lithium is not associated with the other two to form a single group, but is the first electron of the L group, indeed, the first electron of the sub-group LI. As Be, is bound on with a greater energy than Li, the fourth electron of Be seems to be associated with the third electron to form the completed sub-group L_I. The dip in the curve from Be (4) to B (5) seems to indicate that the outer electron of B(5) forms the beginning of a new sub-group LII. seen from the figure this sub-group is completed with N (7), not C (6) as Stoner suggests. O (8) shows the beginning of a new sub-group LIII, which is completed at Ne (10). The comparatively large dip between Ne (10) and Na (11) indicates the beginning of a new principal group, the M group. Although the data are incomplete in the range Na (11) to A (18), enough values are known to make it very probable that these eight electrons are bound on in the same way as the electrons from Li (3) to Ne (10). The proposed electron distribution is therefore M_I, 2; M_{II}, 3; M_{III}, 3. The comparatively large dip in the curve between A(18) and K (19) indicates, according to Bohr *, the beginning of the N group, the completion of the M group being made in some unknown manner before atomic number 29. The first eight electrons of the N group proper, beginning at Cu (29) and ending at Kr (36), seem to be bound on with this same distribution, as is shown in fig. 2. This is not entirely conclusive, as the ionization potentials for atomic numbers 32 and 34 are lacking. However, the evidence is perhaps sufficient to propose the distribution N_I, 2; N_{II}, 3; N_{III}, 3. Fig. 2 also shows the beginning of the O group at Ag (47). With the same type of data as exhibited for the N group the indicated distribution is O_I, 2; O_{II}, 3; O_{III}, 3. shows the beginning of the P group at Au (79). The ionization potential of Bi (83) is given as 8+0.5 volt, indicating a rather large error of measurement. For reasons of symmetry I propose tentatively the distribution Pr. 2: P₁₁, 3; P₁₁₁, 3.

^{* &#}x27;Theory of Spectra and Atomic Constitution,' Bohr.

Table I. shows the data and sources for the first ionization potentials used in plotting figs. 1 and 2.

TABLE I.

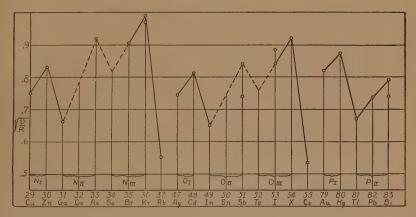
_		1		
At. no.	Term.	ν.	Volts.	Source.
1 H			13.539	(2)
2 He			24.5	(2)
3 Li		1	5.371	(1)
4 Be			9.50	(1)
5B		1	8:34	(1)
6 C			11.3	(1)
7 N			14.494	(1)
80			13.565	(1)
9 F		1	16.9	(1)
10 Ne	1.0	41440.50	21.482	(1)
11 Na	$\frac{1}{1}$ S $\frac{1}{1}$ S	41448·59 61663·0		(2)
12 Mg		48280		(2)
13 Al 14 Si	${}^{2}P_{0}$	64275		(2) (2)
14 Si 15 P	L 0	04270	13.3	(3)
16 S	3P_2	83554	100	(2)
18 A	- 2	COOOT	15.4	(2)
19 K	18	35005.88	101	(2)
20 Ca	1 18	49304.8		(2)
29 Cu	18	62305.86		(2)
30 Zn	1 1S	75758.6	1	$(\overline{2})$
31 Ga	2 P,	48378		$(\overline{2})$
32 Ge	•		7.85	(4)
33 As			11·54±·5	(2)
35 Br			10	(3)
36 Kr			∫ 13·3	(2)
			127	(2)
37 Rb	18	33684.80		(2)
38 Sr	1 1S	45924.31		(2)
47 Ag	18	61093.48	ļ.	(2)
48 Cd	1 1S	72532-8		(2)
49 In	$2 P_1$	46667	7.97 1.05	(2)
50 Sn			7.37 + .05	(5)
			8.5+1.0	(2)
53 1 54 X			10·1±·5 11·5	(3)
55 Cs	18	31406.70	11.9	(2)
56 Ba	1 ¹ S	42029.5		(2)
79 Au	18	74461		(2)
80 Hg	1 18	84181.5		$\binom{2}{2}$
81 Tl	2 P,	49263		(2)
82 Pb	$^{3}P_{0}^{1}$	59821		(2)
83 Bi	-0	00000	8+.5	(2)
				(-)

Millikan and Bowen, Phil. Mag. iv. p. 561 (1927).
 Anregung von Quantensprüngen durch Stösse, Franck and Jordan.
 'The Origin of Spectra,' Foote and Mohler.
 C. W. Gartlein, Phys. Rev. xxxi. p. 782 (1928).
 Green and Loring, Phys. Rev. xxx. p. 574 (1927).

III.

Experimentally 5 absorption limits are known in the M group and 7 in the N group, which would indicate at least 5 sub-levels in the M group and 7 in the N group. As there are 18 electrons in the M group and 32 in the N group, and we have accounted for 8 in each group, we have yet to account for 10 in the M group and 24 in the N group. The formation 2, 3, 3 seems to form a particularly stable set of sub-groups, so that we would not expect the other electrons to intrude on these first three sub-groups as the group is completed. It is proposed tentatively that the M and N

Fig. 2*.



groups are completed as shown in Table II. In Tables II. and III. the arrangements of electrons which are not supported by direct evidence are enclosed in brackets.

TABLE II.

	Sub-groups.							
	ī.	II.	III.	IV.	ν.	VI.	VII.	
K	2							
L	2	3	3					
м	2	3	3	[5	5]			
N	2	3	3	[5	5	7	7	

^{*} Since this paper was written I have found references to the ionization potentials of Ge (32) and Su (50). [See Table I.] The values of $\sqrt{\nu/R}$ are 0.76 for Ge (32) and 0.74 for Sn (50). It is seen that these values lie close to the assumed values shown in fig. 2.

Table III. shows a classification of the electron levels in a few atoms, using Bohr's general scheme together with the distribution of electrons proposed in this paper.

TABLE III.

																						_	_	-
'At. no.	K.		L.]	М					I	N.),		_	F	· .	
At. Ho.	D.	Ι.	II.	ш.	I.	11. 1	11. 1	٧. ٦	v.	ī.	11. 1	II. I	v. 1	7. V	71. V	11.	I. I	I. II	II. I	v. 1	7.	r. 1	1. 11	Ι.
1 H 2 He 3 Li 4 Be 5 B 6 C 7 N 8 O 9 F 10 Ne 11 Na 12 Mg 13 Al	1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	1 2 2 2 2 2 2 2 2 2 2 2 2 2 2	1 2 3 3 3 3 3 3 3 3	1 2 3 3 3 3 3	1 2 2	1																		
18 A 19 K 20 Ca	2 2 2	2 2 2	3 3	3 3	2 2 2	3 3 3	3 3 3			1 2						i								
29 Cu 30 Zn	$\begin{vmatrix} 2 \\ 2 \end{vmatrix}$	2 2	3	3	$\frac{2}{2}$	3	3	[5 5	5] 5	$\frac{1}{2}$														
36 Kr 37 Rb 38 Sr	2	2 2 2	3 3	3 3	2 2 2	3 3 3	3 3	5 5 5	5 5 5	2 2 2	3 3	3 3					1 2				,			
47 Ag 48 Cd	$\begin{vmatrix} 2\\2 \end{vmatrix}$	2 2	3	3	2 2	3	3	5 5	5 5	2 2	3	3	[5 5	5] 5		1	1 2							
54 X 55 Cs 56 Ba	. 2	2 2 2	3 3 3	3 3 3	2 2 2	3 3	3 3	5 6 5	5 5 5	2 2 2	3 3 3	3 3	5 5 5	5 5 5			2 2 2	3 3	3 3			1 2		
79 Au 80 Hg 81 Tl .	. 2	2 2 2	3 3 3	3 3	2 2 2	3 3	3 3	5 5 5	5 5 5	2 2 2	3 3	3 3	5 5 5	5 5 5	[7 7 7	7] 7	2 2 2	3 3	3 3	[5 5 5	5] 5 5	$\frac{1}{2}$	1	
86 Em.	. 2	2	3	3	2	3	3	5	5	2	3	3	5	5	7	7	2	3	3	5	5	2	3	3

Cincinnati, March 17th, 1928. LXI. Studies in the Formation of Kundt's Tube Dust Figures.—Parts I. and II. By Eric J. Irons, B.Sc., East London College, E.1.*

[Plates IX.-XIII.]

Part I.—Dust Figures in General.

§ 1. Introductory and Historical. § 2. Experimental Procedure.

§ 3. Description and Consideration of Plates.

Part II.-Formation of Striæ.

§4. Previous Theoretical and Experimental Work. §5. Present Experimental Work and Conclusions.

PART I.

§ 1. Introductory and Historical.

A SEARCH of the more common text-books (1) on sound, and an examination of the most likely papers (2,3,&4) failed to reveal any systematic photographic record (6) of the dust figures produced in Kundt's tube. It is the principal object of the present paper to rectify this omission by the aid of an apparatus (already described) which allows such

figures to be easily formed.

Kundt (6) performed his original experiments with a fourfoot tube of diameter 3 inch, strewn inside with lycopodium powder. The tube was excited into longitudinal vibration by stroking with a damped woollen cloth, and, when it was closed and suitably clamped, the lycopodium formed figures resembling those of fig. 2 (a) (Pl. IX.) of the present paper. Kundt found that on further stroking the tube the antinodes preserved their places, but that the striations were liable to a small change; on stronger excitation the striæ disappeared and the lycopodium gathered at the nodes. If the dust were evenly distributed on the walls, Kundt obtained, after rubbing, circles of dust on the bottom of the tube around the nodal points of the air column. These he attributed in part to the oscillation of the glass tube itself because of their modified appearance at the nodes of the tube (i.e., the points at which it was clamped). His next experiments were performed in the manner now generally adopted with an independent rod, a metre in length, as a source of vibration. When the length of the air column was suitably adjusted he

^{*} Communicated by Prof. C. H. Lees, F.R.S.

noted that the nodes in the air were very sharply defined by heaps at the bottom and also by vertical rings circling the tube and perpendicular to its axis. In these instances both the end of the metre rod and the stopper in the farther end of the tube determined nodes in the vibrating air, i.e., the length of the column was an integral number of half wave-lengths. If from such a position the stop were moved through a quarter of a wave-length and the experiment repeated then it was found that the nodes preserved their positions with respect to the stop, and that the end of the vibrating rod was situated at an antinode. In these instances the nodal heaps and vertical rings circling the tube were replaced by circles of dust on the bottom of the tube-indicating probably that there was not sufficient energy present to drive the dust to its former positions. Kundt differentiated between the behaviour of lycopodium and of sand in the tube in that firstly, whatever the length of the air column the intensity of the vibration was sufficient to move lycopodium, whereas sand was only affected when the length was an integral number of half wave-lengths; and secondly, the sand striæ were sharper and more regular than those of lycopodium. Iron filings were found to behave in a manner similar to sand. Kundt was not successful in using vapour in place of dust as an indicator. He found that ammonium chloride disappeared at the first stroke of the rod and was precipitated at the nodes in the solid form. He drew the important conclusion that, whatever the form of the figures, the half wave-length of the sound in the tube remained constant.

Other papers of note—illustrated by line drawings—are those of Dvořák (7 &8) in which he offers explanations for many of the phenomena described later in this paper. He also had somewhat narrower tubes than those used in the present work, which were about 4.8 cm. in diameter. He found that when water was placed in the bottom of a tube in which the air was made to resound in the usual manner, it formed a wall across the tube at the antinodes. Assuming that the amplitude of the aerial motion is not small compared with the wave-length, Dvorák shows that there will always be a positive pressure at the nodes, and it is to this pressure that he attributes the motion of the water to the antinodes. He also points out that, owing to friction, the displacement at the middle of the tube must be much greater than at the walls, and suggests that heat is generated at the walls near antinodes giving rise to an increase in pressure at those o ints. From these considerations he concludes that along the walls of the tube air streams from antinodes to nodes and in the interior from nodes to antinodes. Dvorák's experimental evidence for this and his other conclusions will be considered in § 3; but mention may be made here of a confirmation of this idea made a few years after by the late Lord Rayleigh (9). He demonstrated mathematically the existence of a current in a Kundt's tube from antinode to node which is limited to an annulus, contiguous to the walls, of thickness 0.293 times the radius of the tube; and a return current from node to antinode occupying the interior and therefore of diameter 1.414 times this quantity. This vortical motion follows as a consequence of postulating viscosity and taking into consideration second order velocity terms; it does not however depend upon the development of heat.

§ 2. Experimental Procedure.

The essential features of the apparatus used have already been described (10) and are portrayed in fig. 15 (Pl. XII.) (11). As a source of vibration a two-metre rod of brass was employed. This rod, when clamped at its centre and stroked by a resined duster, was capable of emitting both its fundamental and first overtone giving rise to disturbances in the tube having half wave-lengths of c. 19.8 and 6.6 cm. respectively. For most of the work to be described the overtone (generated when the rod is stroked near its centre) was employed, as it was convenient both in regard to its intensity and wave-length. Any trace of the fundamental (12) (obtained by stroking the rod near its end) was sufficient to destroy the figures obtained with the overtone and vice versa, so that the results recorded are not in any way due to the "dual note" property of the rod. It was found unnecessary to take any but rudimentary precautions as to the dryness of the glass tubes which had internal diameters of c. 4.8 cm. The dust to be used was introduced into the tube on a metre scale upon which it had been previously laid in a thin line. The formation of the figures was facilitated by rotating the tube through some 45° to 60° about its axis so that the dust rested on a sloping surface of glass. The distance required between the end of the vibrating rod and the stop for maximum disturbance was determined by trial before a series of photographs was commenced.

The tube containing the figures was photographed in situ from above. The stop was removed from the tube to enable the figures to be illuminated from the end and a black (18) background was placed under the tube, which was completely

shielded from extraneous light. In this way the structure of the figures is emphasized by the shadows cast, and reflexions in the tube are eliminated.

§ 3. Description and Consideration of Plates.

In the Plates the horizontal arrows point towards the end of the vibrating rod used as a source of sound, and the vertical arrows to the actual position of the end of the vibrating rod in the tube in those instances (figs. 11 & 12, Pl. XI.) where, to illustrate the point at issue, it was necessary to remove the rod during the photographic exposure. In interpreting the figures it should be remembered that they were illuminated, as has already been mentioned, by light travelling in the direction of the horizontal arrows.

The substances used to form figures were cork dust, lycopodium, silver sand, magnetized and demagnetized iron filings, pith dust (14), and Kieselsäure (15). With the exceptions of lycopodium and Kieselsäure these powders were obtained in a sufficiently divided state by filtering through a copper gauze having some 75 meshes to the inch. The iron filings were demagnetized by heating. All the figures described in this section were obtained with the overtone of half wavelength c. 6.6 cm.

Figs. 1, 2, & 3 (Pl. IX.) show typical results for cork, lycopodium, and sand respectively obtained by a single stroke of the rod (a), by two or three hard strokes (b), and by continued stroking (c). By inclining the tube until its axis was vertical most of the powder fell out, but that which clung to the tube emphasized some essential features when photo-

graphed (vide (d)).

In (a) the formation of striæ at the antinodes is shown: these will be considered in detail in Part II. On further excitation these strix are destroyed and the dust travels towards the nodes—most of it going to the node farthest from the vibrating rod—to form, in the case of light powders, a heap and a double ring (16) which encircles the tube (17) ((c) and (d)). These rings are not so evident with sand, which is more difficult to move. Figs. 3 (b) & 3 (c) (Pl. IX.) for sand resemble Dvorák's drawings for lycopodium. He notes that for stronger strokes the limits g and g' separate farther and farther and approach the nodes which, however, they never reach, because as the note gets softer they retreat almost instantaneously to their old positions. This effect has not been found troublesome with lycopodium in the present work, and Dvořák claims to have been more

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successful in driving g and g' to the nodes by using an

extremely small quantity of the seed.

The relative positions of antinodes, rings and nodes may be seen from Table I., which gives an analysis of the observations made in a typical instance.

TT	6	-
TABLE		
TUDLE	Z.	50

$\lambda/2$.	Antinodes.	Nodal Rings.	Nodal Heaps.	λ/2.
	Position	of end of vibrating r	od 0·0	
	(Not measurab	le.)		
	1	6.4	6.5	
0.0	10.1	.7·1		6.6
6.6	16.7	13.1	13.1	
6.5	10,1	13·6 19·8	19.7	6 .6
00	23.2	20.3	101	6.5
6.5	}	26.4	· 26·2	
	29.7	26.9		6.6
6.5	1 00.0	32.8	32.8	0.0
6.6	36:2	. 33•5 39•3	39.4	6.6
00	42.8	40'0	20 x	6.6
6.5	{	46.0	46.0	00
	49.3	46.6	}	6.6
6.6	}	52.6	52.6	
	55.9	53.2	59.2	6.6
			99.2	

Position of stop...... 59.2

In this instance the end of the vibrating rod coincided with a node in the gas column; in general the nearest node

is in front of this position (18) (i. e., nearer the stop).

The figures formed by demagnetized and magnetized iron filings are shown in figs. 4 & 5 (Pl. X.) respectively. Strike corresponding to fig. 4 (a) (Pl. X.) did not appear to be formed with magnetized iron filings, which seemed to slip immediately to the bottom of the tube, moving towards (fig. 5 (a) (Pl. X.)), and soon surrounding (fig. 5 (b) (Pl. X.)) the nodes. A close examination of fig. 5 (a) (Pl. X.), however, reveals a certain amount of striation similar to that shown in fig. 4 (b) (Pl. X.). The significance of this diversity of behaviour will be discussed in Part II. of this paper. Figs. 4 (c) and 4 (c') (Pl. X.) show two further stages of the "closing-in" process.

Figs. 6 (a) and 6 (b) (11. X.) show the initial and a later

stage of the process using pith dust.

The figures obtained with *Kieselsäure* are shown in fig. 7 (Pl. X.) (the (a), (b), and (c) correspond to the usual stages).

This substance being very light and finely divided tends to spread in the tube rather than to form into figures. Fig. 7 (c) (Pl. X.) illustrates this point, and fig. 7 (c') (Pl. X.) is a deeper print from the same plate to show that

the figures are not devoid of structure.

In order to minimize as far as possible the effect of gravitation on the motion of the particles, the figures were next obtained on a plane black lath placed horizontally along the axis of the tube. The results for cork, lycopodium, and sand are shown in figs. 8, 9, and 10 (Pl. XI.) respectively, and in the originals were remarkable for their beauty. The photographs of cork and lycopodium show the results of increasingly strong excitation: nothing more of interest was obtained with sand. As the striæ were irregular and showed a marked tendency to bifurcate or intermingle the device

was not subsequently used.

During the progress of this work it became increasingly evident not only that dust was carried from antinode to node but also that it shifted along the tube from the vibrating rod towards the stop. The experiments with cork and lycopodium recorded in figs. 11 and 12 (Pl. XI.) respectively illustrate this observation. A line of dust was introduced into the tube in which the position of the end of the vibrating rod is indicated by a vertical arrow (figs. 11 (a) and 12 (a)) (Pl. XI.); the stop being towards the right end of the photographs. In the first instance the dust extended from the arrow in both directions; in the second it was confined to the resonant air column. On the first stroke of the rod striæ were formed (figs. 11 (b) and 12 (b), Pl. XI.), and with a few hard strokes the characteristic figures, already described, were developed in the sound-tube (figs. 11(c) and 12(c), Pl. XI.). The rapidity of progress of the cork dust to the left in fig. 11(c) (Pl. XI.) should be noted: the end of the tube to which it approached was open. From fig. 11 (d) (Pl. XI.) it will be seen that further stroking has removed most of the dust from the neighbourhood of the end of the vibrating rod, and that the tongue of dust shown in fig. 11(c) (Pl. XI.) has moved towards the stop to form another node. Figs. 11 (e) and 12(d) (Pl. XI.) register further progress, and the lengths of the dust columns here shown should be compared with the corresponding lengths of undisturbed dust in the (a) photographs. It was afterwards found that this effect is even more strongly developed by the use of Kieselsäure, but it was not obtained with sand. Dvorák observed the existence of this shift, and stated that it was quicker nearer

the source of vibration. To verify this statement two 10 cm. lengths of cork dust were placed in a tube with those ends which were nearer the rod coincident with nodes of the air column. After the rod was rubbed the lengths of dust nearer to, and farther from, the rod were found to have increased to 22 and 18 cm. respectively. Dvořák's observation that the dust moves quickest in passing an antinode was confirmed. He found with an open tube that the dust has a tendency to return from the end and remain in the penultimate quarter wave-length, although each stroke of the rod causes it to emerge in a small cloud; this observation was also substantiated. The shift itself together with its relative magnitudes near the two ends of the air column may be explained in terms of the inequality of intensity which exists between the incident and reflected trains constituting the stationary wave in consequence of losses on reflexion from the end of the tube and the damping effect of the walls.

Fig. 13 (Pl. XII.) shows that the effects so far demonstrated are not localized in any particular part of the tube. Nine half wave-lengths were contained in the sound column, the powder used was cork dust, and the photographs (a), (b), and (c) correspond to stages already described with reference to fig. 1 (Pl. IX.). The shift of dust towards the stop is clearly indicated in fig. 13 (c) (Pl. XII.), and by inspection of fig. 13 (a) (Pl. XII.) it will be seen that nodes and antinodes are everywhere well defined in the first stage. This result, contrary to Dvorák's observation that strize were formed everywhere near the vibrating rod although somewhat thicker near the nodes, has also been obtained with much longer resonant columns. It sometimes happens, however, that the antinode figure adjacent to the end of the vibrating rod is not well formed, as, for example, in the instance recorded in Table I.

It remains to discuss the vortical phenomena observed by Dvorak when experimenting with Kieselsäure. On excitation this substance was found to rise to the interior of the tube and there to proceed from node to antinode, where it formed a wall across the tube. On the cessation of the source of sound each wall was found to divide into two repelling parts each of which formed a cloud at the nodes with the neighbouring halves from the adjacent loops. An attempt was made by Dvorak to verify the motion described by noting the behaviour of a small rectangle of paper suspended in the tube; this introduced a disturbing influence, however, which gave rise to anomalous results (19).

Now the motion of *Kieselsäure* in a strongly excited Kundt's tube is not easy to follow with the unaided eye and, in consequence, Part III. of this investigation will be concerned with the results of a cinematographic study (at present in active progress) together with some general conclusions from all the observations.

The photographs of the present paper-notably those for Kieselsäure—vield some indication of the existence of a current in the centre of the tube from node to antinode. In fig. 7(c') (Pl. X.) the antinodes are marked almost as definitely as the nodes and it would appear, both with reference to this figure and also figs. 2(c) and (d) (Pl. IX.) for lycopodium, that with light powders a cloud occurs near the antinodes which might well be due to the powder falling diffusely from the centre of the tube on the cessation of the exciting vibration, although according to Dvořák the powder falls back to the nodes. It is hoped to settle this point by cinematographic observations, and in Part III. the connexion between this effect and the "line density" and size of the dust particles will also be discussed. There is a slight suggestion of the phenomenon with cork and pith figures but none with sand and iron filings. That these clouds occur slightly towards the "stop" side of the antinodes is ascribed to the general shift towards the stop already mentioned.

Figs. 14 (a) and (b) (Pl. XII.), obtained by the method described in § 5 with cork dust and sand respectively, show a pronounced curvature of the striæ concave towards the nodes. This effect was noted by Dvorák, who stated that it appeared as if two systems of striæ were travelling in opposite directions through one another. Dvorák considered that the curvature was due to the thin layers of dust near the walls of the tube going from antinodes to nodes, while the thicker layers at the bottom of the tube, which were farther removed from the influence of the tube's surface, travelled in the opposite direction. It is notable that these circular striæ are obtained when the excitation of the air is greater than that required to give straight striæ, but not so great as to give the effects shown in "(b)" photographs. (It often happens (vide figs. 1 (b) and (c), 3 (b) and (c) (Pl. IX.), and 6 (b) (Pl. X.)) that thick circular strize occur in "(b)" photographs; they appear to be formed as the exciting note dies down.)

It appears probable, both from the manner of its ejection and the fact that the "wall stream" is towards the vibrating rod, that dust expelled from the end of an open tube is carried by the "central stream."

PART II.

§ 4. Previous Theoretical and Experimental Work.

The formation of striæ in a Kundt's tube has been explained by König $^{(20)}$ in terms of the hydrodynamical forces between spheres situated in a vibrating fluid. Starting from Kirchhoff's general equations $^{(21)}$ for the motion of two spheres in a perfect fluid, König showed that if R_1 and R_2 be the radii of two spheres separated by a distance r_0 , then the components of the forces between them are

$$\begin{split} {}^{1}X &= -\left(\frac{3}{2}\pi\rho R_{1}{}^{3}R_{2}{}^{3}\omega_{0}{}^{2}/r_{0}{}^{4}\right)\sin\theta(1-5\cos^{2}\theta), \\ Z &= -\left(\frac{3}{2}\pi\rho R_{1}{}^{3}R_{2}{}^{3}\omega_{0}{}^{2}/r_{0}{}^{4}\right)\cos\theta(3-5\cos^{2}\theta), \end{split}$$

 ω_0 being the velocity of the stream (considered parallel to the axis of z), ρ the density of the medium, and θ the angle which the line of centres of the spheres makes with the z axis. Further, as he considered the motion to be in the xz plane (i. e., confined to the bottom of the tube), König wrote

$$Y=0.$$

From these equations it follows that, n being an integer,

when $\theta = n\pi$ $X = 0, Z = 3\pi \rho R_1^3 R_2^3 \omega_0^2 / r_0^4,$

and when $\theta = n \frac{\pi}{2}$ $X = -\frac{3}{2}\pi \rho R_1^3 R_2^3 \omega_0^2 / r_0^4$, Z = 0.

Thus it appears that spheres situated in a line parallel to the axis of the tube (i.e., in line with the direction of propagation of the sound disturbance) repel, while those perpendicular to the direction of the tube attract each other. Ignoring other possibilities the raison d'être for the formation of striæ is suggested.

Cook (22) attempted to verify these formulæ experimentally and concluded that "A perfect fluid contains forces which are essential but are not sufficient to produce laminæ in the form in which they exist" and that the necessary new forces are "most probably due to the viscosity of the medium." By the microscopical examination of the movements of powder particles, and subsequently of small spheres (23) suspended in a Kundt's tube, Cook found in addition:—

1. That particles with their line of centres transverse to the stream lines were maintained in equilibrium when they were not more than half their diameter apart. (König's theory requires that they should be in contact.)

2. Particles with their line of centres along the stream lines and within half their own diameter approached, acted as a single system and, like a Rayleigh disk, rotated into a

plane perpendicular to the stream lines.

Pointing out that ω_0 varies along the tube when stationary waves are set up so that the arrangement of particles should not be the same in all parts of the tube, Robinson (24) has devised a test of König's theory based upon the measurement of inter-strize distances. Any one particle in a lamina is in equilibrium under the action of the repulsive forces due to all other particles in the laminæ to either side of it. Summing only the effects of the particles constituting the first laminæ to the right and to the left of the lamina under consideration, Robinson showed that $a_{r,r+1}/a_{1,2} = \cos^{4/3}(\frac{1}{2}\pi k/l)$, where $a_{r,r+1}$ represents the distance between the rth and (r+1)th ripples distant k from an antinode and l is the quarter wave-length of the sound disturbance. To test this Robinson performed some experiments with sources of vibration yielding half wave-lengths of 3.63 and 5.03 cm. respectively in the sound tube and succeeded in showing that the distance between the ripples was greater at the antinodes than at the nodes.

În a later paper Robinson (25) deduced König's theory on different grounds—though still basing the analysis on Kirchhoff's equations—and extended it to the case where the two spheres do not experience the same velocity. He then found that the square of the mean velocity (ω_0^2) should be replaced by the product of the separate velocities in the formulæ quoted, and finally obtained $a_{r,r+1}/a_{1,2} = \cos^{2/3}(\frac{1}{2}\pi k/l)$ as the expression for inter-striæ distances. By experiment the ratio of the distances between the ripples near the nodes and at the antinodes for various combinations of frequency and powder was found to vary between 0.94 and 0.66, and the mean of a number of observations gave 0.78. The ratio of the mean ordinates of the curve $y = b \cos^{2/3} \phi$ for values of the abscisse corresponding to the positions of the strice distances measured was also obtained: the process was repeated for the curve $y = b \cos^{4/3} \phi$ and the three sets of ratios were compared. Assuming the striæ to extend 7/9 of the distance from an antinode to a node—as Robinson's observations led him to believe was generally the case—the ratios for the 2/3 and 4/3 power formulæ respectively are 0.76 and 0.56, and on these facts quantitative agreement between the $\cos^{2/3}(\frac{1}{2}\pi k/l)$ formula and the experimental result of 0.78 was claimed.

In a further paper Robinson (26) has shown how ripple may be formed from a powder uniformly distributed on th bottom of a tube by hydrodynamical forces without the intervention of viscous forces. As he remarks, however, this does not preclude the possibility that viscosity may be a facto affecting the formation of ripples.

§ 5. Present Experimental Work and Conclusions.

Some observations and measurements on ripples appeared to form the natural sequence to those observations already recorded in the previous paragraphs and accordingly some dozen photographs, with different magnifications, of ripples formed by cork dust, sand, and *Kieselsäure*, in conjunction with notes giving rise to half wave-lengths in the tube of c. 6·6, 7·7, 10·8, and 19·8 cm. respectively were taken. They are shown in figs. 19, 18, 17 (Pl. XIII.), and 16 (Pl. XII.)

For this purpose it was deemed better to use transmitted light and accordingly the figures were photographed from above when illuminated from below by a system of frosted glass electric light bulbs shining through two sheets of opal

glass.

The figures were produced from a thin line of dust in the usual manner except that, to minimize the effect of gravitation, the tube was not rotated about its axis, so that the dust lay along the bottom of the tube. So long as the "line density" of the dust be constant its initial distribution is immaterial as, on the completion of a stroke of the exciting rod by a resined cloth, the dust particles must have been established in positions of equilibrium. For the formation of striæ it is necessary that the intensity of the exciting note should be sufficient to agitate the dust, but not so great as to project it towards the nodes.

From an examination of figs. 16 (Pl. XII.), 17, 18, and 19 (Pl. XIII.) it will be seen that thick and thin, or, as it is perhaps preferable to call them, major and minor striations alternate in the dust figures and that these alternations are especially marked in the neighbourhood of the antinodes. So far as the author is aware this effect has not before been noted; it is particularly prominent when the tube is in resonance.

During the process of stroking the striæ may be observed to move slightly backwards and forwards in a direction parallel to the axis of the tube (vide Kundt's observation) and in so doing the minor ripples may be drawn into and absorbed by the major and be reformed later. Now according to Robinson's theory of ripple formation (26)

there is always a ripple at an antinode: in practice this is not a general rule and it is suggested that although initially the antinode may be defined by a ripple, yet, due to the backward and forward motion, at the conclusion of a stroke of the roll the ripple may be displaced from the antinodal

position.

Despite the presence of minor ripples of which Robinson's theory takes no account, it was thought well to measure the distances between the major strize of the figures of Pls. XII. and XIII., and accordingly each photographic negative was illuminated from below and the distances read off with a travelling microscope. To keep the relation between $a_{r,r+1}$ and k as general as possible, let us write

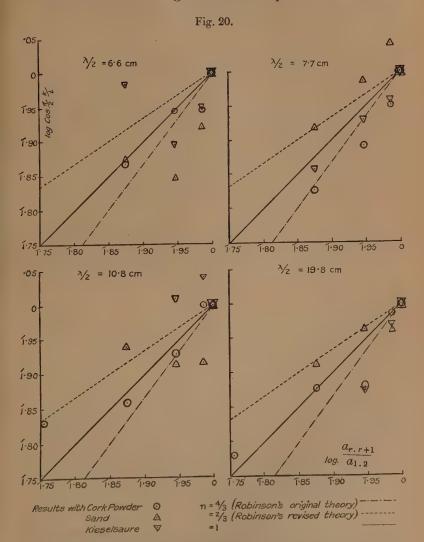
$$a_{r,r+1}/a_{1,2} = \cos^n \left(\frac{1}{2}\pi k/l\right),$$

so that on plotting $\log a_{r,r+1}/a_{1,2}$ against $\log \cos (\frac{1}{2}\pi k/l)$ a straight line should result from which the value of n may be obtained $^{(27)}$.

The experimental results were too irregular to justify plotting each value of $\log a_{r,r+1}/a_{1,2}$ against the corresponding value of $\log \cos\left(\frac{1}{2}\pi k/l\right)$. In consequence the distance between two nodes was divided into thirteen equal sections, and the mean distance between major striæ in the middle section was taken as $a_{1,2}$. The mean distance between major striæ in the sections to the immediate right and left of the middle section was divided by $a_{1,2}$ and the logarithm of the quotient was plotted against $\log \cos\left(\frac{1}{2}\pi \cdot 2/13\right)$ and so on. The resulting points are shown in fig. 20, to which lines indicating values of n equal to 4/3 (Robinson's original theory), to 2/3 (Robinson's revised theory), and to unity have been added.

Cork dust gives the most consistent results with a value of n equal to unity. In most instances it is almost impossible to assign a numerical value to n, and perhaps the only conclusion justified by these measurements is expressed by saying that, in general, the distance between striæ increases on approaching an antinode. It would perhaps be better to regard this work as failing to support Robinson's theory rather than disproving it. In work of this nature it is almost impossible to ensure that experimental conditions are such as to justify the assumptions upon which the theory is based. Moreover the presence of minor striæ immediately vitiates the theory and it is not easy to see how to introduce a correction for them, as minor striations consisting of single strings of particles have been observed with sand, while in other instances minor striæ comparable in size with major striæ have occurred.

It remains to remark on the fact (noted under § 3) that iron filings appear to form striæ more readily when demagnetized than when magnetized. It is probable that when



magnetized the filings form into chains the mutual attractions between the members of which neutralize to some extent the repulsive forces which must exist along the axis of the tube in order that discrete laminæ may be formed. Robinson (28)

has attempted quantitative experiments on these lines but without positive results.

Finally, I wish to express my thanks to Prof. C. H. Lees, F.R.S., both for the facilities he has provided and for the interest he has taken in this work.

References and Notes.

(1) Some photographs of somewhat ill-defined striæ are given in Winkelmann's 'Handbuch,' ii. Akustik, p. 198 (1909).

(2) Photographs of striæ formed in a dust tube by an electric discharge have been published, e. g., in a paper by Richmond, Phil. Mag. ser. 6, xviii. p. 771 (1909). (See Pl. XXIV. of that paper).

(3) Some small-scale photographs are given by Hartmann and Trolle,

Jour. Sci. Instrs. iv. p. 101 (1927).

(4) The application of the Kundt's tube method to the determination of the velocity of sound in liquids is described by Kundt and Lehmann (Pogg. Ann. cliii. p. 1, 1874), and by Dvořák (Sitz. der Akad. der Wiss. zu Wien, lxxi. 2, p. 315, 1875). In the former paper striæ are represented in a conventional manner, and the latter paper includes a woodcut (fig. 7) showing strize formed in water in various circumstances.

(5) This is no doubt due in part to the fact that "Until the last quarter of the 19th century reproductive processes, save as regards line reproduction, can hardly be said to have had an existence."

(Encyc. Brit. xxii. p. 408, 11th edition, Article "Process.")

(6) Kundt, Pogg. Ann. exxvii. p. 497 (1866).

(7) Dvorák, Pogg. Ann. cliii. p. 102 (1874).

(8) Dvorák, Pogg. Ann. clvii. p. 42 (1876).

(9) Rayleigh, Phil. Trans. clxxv. Part I. p. 1 (1884), or Scient. Papers, ii. p. 239. See also 'Sound,' ii. p. 333 (1926).

(10) Irons, Phil. Mag. v. p. 580 (1928).

(11) For the purpose of this photograph the rod was pulled back in its support and the stop was advanced.

(12) The fundamental could be eliminated by holding the rod at a point one-sixth of its length from one end during the stroking.

(13) The tube was white enamelled underneath for the purpose of

photographing the iron filings.

(14) The use of ground sunflower pith has been recommended by R. V. Cook ('Nature,' cxviii. p. 157, 1926), who with a closed tube (dimensions not stated) obtained "disks that extended completely across the tube and having the same diameter as the inside of the tube." In my experiments ground elder pith, which was more readily available, was used. Its behaviour appeared comparable with that of cork dust. Cook also recommends the use of an organ pipe as a source of vibration; this is not new, and by its use, in conjunction with Kieselsaure, Kundt (Pogg. Ann. exxviii. p. 337, 1866) appears to have obtained results similar to those of Cook, leading him to believe that conditions are the same throughout the whole cross-section of the tube.

(15) Kieselsäure is prepared by "the action of water upon the tetrahalides of silicon, the gelatinous hydrate obtained yielding silica on ignition as a very light white powder." (Thorpe's Dict. App. Chem. vi. p. 84. Article "Silicon," 1926.) The lightness of this form of sitica, as compared with sand, is due to the air which pervades its interstices.

(16) The ring nearer the stop is usually faint in comparison with that nearer the vibrating rod, but is readily seen when the tube is

illuminated in the manner described in § 2.

(17) The sharply-defined nodal heaps of fig. 2 (c) would seem to indicate that, where possible, it would be of advantage to revert to the practice of Kundt (Pogg. Ann. exxxv. p. 337, 1868), and determine half wave-lengths by measuring the positions of the centres of these heaps, rather than by noting the positions of corresponding points of striæ figures.

(18) Vide Irons, Ref. (10).

(19) In a continuously excited tube a hot-wire method might with advantage be employed to investigate the motion, and the author hopes to report the result of such an investigation in a later

- paper.
 (20) König, Wied. Ann. xlii. pp. 353, 549 (1891).
 (21) Kirchhoff, Mechanik, 18 and 19 Vorlesungen, or Robinson, Phil. Mag. xix. p. 476 (1910).
- (22) S. R. Cook, Phil. Mag. iii. p. 471 (1902). (23) See also Cook, Phil. Mag. vi. p. 424 (1903). (24) Robinson, Phil. Mag. xviii. p. 180 (1909). (25) Robinson, Phil. Mag. xix. p. 476 (1910). (26) Robinson, Proc. Phys. Soc. xxv. p. 256 (1913).

(27) When k = l, i.e., at a node $\log \cos (\frac{1}{2}\pi k/l)$ is $-\infty$. In practice, however, clear ripples rarely extend farther than 0.81 from an antinode, and over this range the plot proposed is admissible.

(28) Robinson, Phys. Zeit. ix. p. 807 (1908).

LXII. The Effect of a Finite Baffle on the Emission of Sound by a Double Source. By Dr. M. J. O. STRUTT *.

ABSTRACT.

In order to calculate numerically the effect of a finite baffle plate on the sound emitted by a double source (representing an oscillating circular plate of dimensions small with respect to the wave-length), the following model is adopted. Two simple sources of equal amplitude and opposite phase are placed in two diametral points of a sphere. The effect of increasing the diameter of this sphere on the circulation of air from one source to the other is taken to be essentially the same as that of a flat baffle plate of finite dimensions. The total energy emitted is calculated as a function of the ratio of the sphere's circumference to the wave-length, and the result is shown in Graph II. The effect of the baffle on the directive properties of the emitter is exhibited in

^{*} Communicated by the Author.

three graphs as a function of the ratio quoted above. For comparison, the energy emitted by two equal sources of opposite phase without baffling sphere is calculated and shown in Graph VI. This tends to strengthen the hypothesis that the sphere may well represent the effect of a flat baffle.

Introduction.

A FLAT membrane oscillating free in air will only radiate little energy as long as its dimensions are small compared with the wave-length of the sound emitted. This is mainly due to the circulation of air from one side to the other, thus levelling the differences of pressure and destroying the sound-waves. Messrs. Rice and Kellogg * pointed out that this effect explains why a good loud-speaker without baffle emits only little energy in the low notes. The use of a baffle-plate diminishes the circulation of air and the levelling of pressure differences, and hence helps to bring out low notes from the loudspeaker. It is clear that an infinite baffle fulfils this aim in the most perfect way, but at the same time shuts off one-half of the loudspeaker.

Practically only finite baffles are used, and so the question arises as to what dimensions should be given to the baffle. From physical reasoning we may expect that enlargement of the baffle beyond a certain critical point, the wave-length being given, will only have little effect, as the conditions for an infinite baffle are practically reached. From considerations of phase, one would expect that this point should be about where the shortest air-path between the front and the back of the loudspeaker equals one-half of the wave-length.

The straightforward way mathematically would be to consider a very oblate spheroid with two equal sources of opposite phase on the ends of the smallest diameter. It appears, however, that spheroidal harmonic analysis has not been carried far enough numerically to effect calculations with this model for any dimensions comparable with the wavelength.

The model adopted therefore consists of a sphere with two equal sources of opposite phase at the ends of one diameter. Numerical calculations may then be carried far enough to cover all interesting points.

cover all interesting points.

^{*} Journ. Am. Inst. El. Eng. p. 982 (1925).

Total Emission of Energy.

If we have a sphere, on which sources of sound are distributed symmetrically with respect to a diameter, the amplitude of velocity on the sphere being given by a function $U(\vartheta)$, where ϑ is the angle with the diameter of symmetry, the velocity potential at any point is given by

$$\psi = -\frac{c^{2}}{r} \cdot \exp\{ik(dt - r + c)\} \cdot \sum_{n=0}^{\infty} \frac{f_{n}(ikr)}{F_{n}(ikc)}$$
$$\cdot (n + \frac{1}{2}) \cdot P_{n}(\mu) \cdot \int_{-1}^{+1} UP_{n}(\mu) d\mu, \quad . \quad (1)$$

where

 ψ = velocity potential, $k = 2\pi/\lambda$, λ = wave-length, c = radius of sphere, d = velocity of sound, t = time, $i = \sqrt{-1}$, r_1 = distance from centre of sphere, P_n = Legendre's function of order n, $\mu = \cos \vartheta$.

For large values of r we have $f_n(ikr)=1$. The functions $F_n(ix)$ are given by

$$\begin{split} \mathbf{F}_{1}(ix) &= 2 + i\left(x - \frac{2}{x}\right), \\ \mathbf{F}_{3}(ix) &= 7 - \frac{60}{x^{2}} + i\left(x - \frac{27}{x} + \frac{60}{x^{3}}\right), \\ \mathbf{F}_{5}(ix) &= 16 - \frac{735}{x^{2}} + \frac{5670}{x^{4}} + i\left(x - \frac{135}{x} + \frac{2625}{x^{3}} - \frac{5670}{x^{5}}\right), \\ \mathbf{F}_{7}(ix) &= 29 - \frac{4284}{x^{\frac{3}{2}}x^{2}} + \frac{148995}{x^{4}} - \frac{1081080}{x^{6}} \\ &+ i\left(x - \frac{434}{x} + \frac{29925}{x^{3}} - \frac{509355}{x^{5}} + \frac{1081080}{x^{7}}\right). \end{split}$$

As will appear shortly, only odd indices are needed. In our model U is only sensible for $\mu = \pm 1$, and in these points U has equal values of opposite sign. Hence the

integral in equation (1) gives zero for any even n, so that we may write

$$\psi = -\frac{1}{\pi r} \exp\{ik(dt - r + c)\} \sum_{n=1,3,5...} \frac{f_n(ikr)}{F_n(ikc)} \cdot (n + \frac{1}{2}) \cdot P_n(\mu) \cdot Q, \quad . \quad . \quad (2)$$

where Q represents the product of the given velocity of air and the element of surface integrated over the surface of one source.

From (2) we may readily calculate the energy emitted by our model and find

$$W = \rho \frac{k^2 dt}{2\pi} \cdot Q^2 \cdot 2e.$$
 (3)

(ρ density of air),

where

$$e = \sum_{n=1,3,5,...} \frac{n+\frac{1}{2}}{\alpha_n^2+\beta_n^2}$$
 and $F_n(ikc) = \alpha_n + i\beta_n$. (4)

So e is a measure for the energy emitted, with frequency and Q kept constant. Numerically we find from (4):

$$ko$$
 $\cdot 5$ $\cdot 8$ $\cdot 1 \cdot 0$ $\cdot 1 \cdot 3$ $\cdot 1 \cdot 5$ $\cdot 2 \cdot 0$ e $\cdot 0925$ $\cdot 218$ $\cdot 300$ $\cdot 369$ $\cdot 372$ $\cdot 344$ kc $\cdot 2 \cdot 2$ $\cdot 2 \cdot 5$ $\cdot 3 \cdot 0$ $\cdot 3 \cdot 5$ $\cdot 4 \cdot 0$ $\cdot 4 \cdot 5$ $\cdot 5 \cdot 0$ $\cdot 10$ e $\cdot 336$ $\cdot 349$ $\cdot 407$ $\cdot 423$ $\cdot 404$ $\cdot 411$ $\cdot 443$ $\cdot 461$

All calculations, starting with the tabulation of the functions F_n , have been carried out with a slide-rule of 25 cm., so that the last figure of the result may be wrong by some units. The tabulated values of the functions $F_n = \alpha_n + i\beta_n$ are given in the appendix. For kc = 5, 1.0, 2.0, and 10 values given by Rayleigh * have been used.

It is desirable to know the value of e for an infinite baffle $(ke \to \infty)$. In this case we have twice the configuration of a simple source close to an infinite plate. Hence, in our notation, the energy emitted is

$$W = 2 \cdot \rho \frac{k^2 dt}{4\pi} \cdot Q^2. \quad . \quad . \quad . \quad . \quad (5)$$

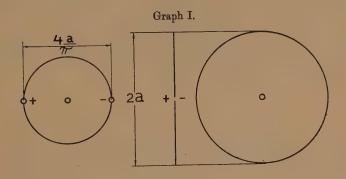
By comparing (3) and (5) we easily deduce that the value of e for $kc \to \infty$ must be one-half.

^{* &#}x27;Theory of Sound,' ii. p. 255 (2nd ed.).

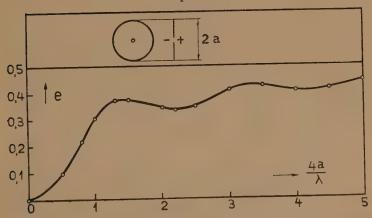
Here c is the radius of the sphere and

$$kc = \frac{2\pi}{\lambda}c$$
.

We may expect that the shortest paths of sound in the spherical model may be compared with the shortest path



Graph II.



in the case of a flat baffle in the shape of a circular ring. Hence we should take (Graph I.)

$$kc = \frac{2\pi c}{\lambda} = \frac{4a}{\lambda},$$

where α is the radius of the outer circumference of the baffle. From the above table Graph II. has been drawn, giving e as a function of $4a/\lambda$.

It appears that enlargement of a beyond $a \sim \lambda/4$ gives only little effect. This may be called the saturation value of a indicated in the introduction. If $4a/\lambda=1$, the shortest path from the front to the back of the vibrating membrane, or in our model from the positive source to the negative one, equals one-half of the wave-length, bearing out what would be expected from phase considerations. It is interesting to see from Graph II. that the value of e with $a=\lambda/4$ is still only 60 per cent. of the value for an infinite baffle. Increasing $4a/\lambda$ to 3 brings e to about 80 per cent. of the final value. The first oscillation in the e-curve is about 10 per cent. of the e-value at that point, and so may be detected by a careful measurement.

Directive Properties.

A flat circular membrane, oscillating as a whole, situated in an infinite baffle, has remarkable directive properties if the dimensions become larger than the wave-length, the radiation diagrams produced being wholly equal to the well-known diffraction patterns obtained with a circular aperture. When the dimensions of the membrane are small compared with the wave-length, all of the directive properties are lost as long as the baffle is infinite, the radiation diagram being a semicircle. Here a finite baffle may step in and cause directive properties, as soon as the dimensions of the baffle are comparable with the wave-length. But instead of a sharp beam with some small companions, as in the case considered first, we can only produce some rather broad beams of similar size.

In order to obtain the radiation diagrams for various values of $4a/\lambda$, we must go back to equation (2). Let F and G be given by

$$F + iG = \sum_{n=1,3,5...} \frac{n + \frac{1}{2}}{F_n(ike)} P_n(\mu), \dots$$
 (6)

then

$$I = F^2 + G^2$$

is a measure for the intensity in the direction $\mu = \cos \vartheta$ considered. I have calculated F and G from (6) for various values of μ , and for $kc = 4a/\lambda = \frac{1}{2}$; 1; 2. In these calculations use has been made of figures given by Rayleigh. The result is exhibited in Table I.:—

TABLE I. $4a/\lambda = \frac{1}{2}$.

		_			
$9 = \cos^{-1} \mu$, 0°.	15°.	3 0°.	`	45°.	60°.
F1811	·1754	·1589		·1312	· 0 928
G3168	·3069	•2780		•2292	·1643
$F^2+G^2=I$. ·1331	·1250	•1024		.0697	•0356
	4a	$/\lambda = 1.$			
$9 = \cos^{-1} \mu$. 0°.	15°.	30°.		45°.	. 60°.
F5540	•5428	.5046		·4324	·3206
G2705	·2 6 62	•2501		·2171	•1631
$F^{\circ}+G^{2}=I$. ·3790	•3660	·3173		•2346	•1297
	$4a_{i}$	$\lambda = 2$.			
$9=\cos^{-1}\mu$. 0°.	15°.	3 0°.	45°.	60°.	75°.
F2737	•3119	· 4 011	·4772	•4552	· 2 847
-G1359	•1554	· 2 117	• 23 88	·2275	·1419
F ² +G ² =I '0934	·1214	.2065	·2806	· 2 593	.1016

The calculations have been carried out with the slide-rule

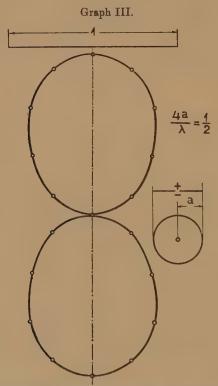
of 25 cm., and so the last figure is uncertain.

From Table I. the radiation diagrams (Graphs III., IV., and V.) have been drawn. If $4a/\lambda = \frac{1}{2}$, the departure from a cosine law, which holds exactly in the limiting case $(4a/\lambda = 0)$, is not very marked. The intensity, being zero in our measure for $4a/\lambda = 0$, is still weak. Increasing $4a/\lambda$ to 1 brings the intensity to a much higher value, at the same time making the departure from the cosine law of radiation conspicuous. For $4a/\lambda = 2$ two distinct beams have been formed. We may expect that with still greater

baffles more distinct beams will occur, the total aspect of the diagrams, however, becoming nearly a circle for large values of $4a/\lambda$.

From Table I. we may readily calculate the phase-angle $\theta = \tan^{-1}(G/F)$ for different directions, which, however,

varies little with 3.

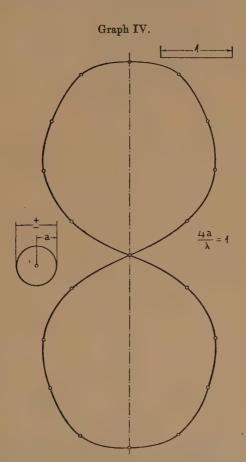


Graphs III., IV., and V. represent the intensity of sound in different directions as influenced by the enlargement of the baffle, and should be rotated about the vertical axis.

Radiation by Two Separate Simple Sources without Baffling Sphere.

The supposition that our spherical model might fairly well represent the effect of a flat baffle, is based on the assumption that the preponderant factor in the emission of energy is the shortest air-path between the front and the back of the vibrating membrane. If this is right, the effect

of increasing the distance between two simple sources of equal amplitude and opposite phase must be essentially the same as that of a baffle, the distance between the sources being the important parameter. Now, by an elementary



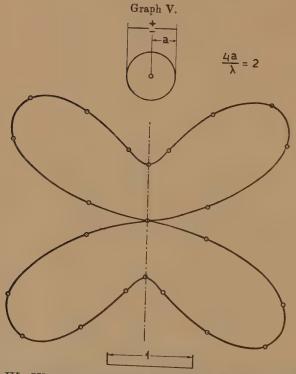
calculation we find for the velocity potential of two such sources of strength Q each:

$$\begin{split} \psi = & -\frac{\mathbf{Q}}{2\pi r} \cdot \exp\left\{ik(dt-r)\right\} \cdot \cos\left(\frac{kd_1}{2}\cos\vartheta\right) \\ & (d_1 \text{ distance between sources}). \end{split}$$

This expression holds good only for large r, r being the

distance from the centre of the line joining the sources. From the above formula we may find the total energy emitted:

$$\begin{split} \mathbf{W} = & \rho \, \frac{k^2 \, dt}{2\pi} \, \mathbf{Q}^2 \Big(1 - \frac{1}{k d_1} \sin \, k d_1 \Big) \\ = & \rho \, \frac{k^2 \, dt}{2\pi} \, \mathbf{Q}^2 \, . \, \mathbf{E}. \end{split}$$



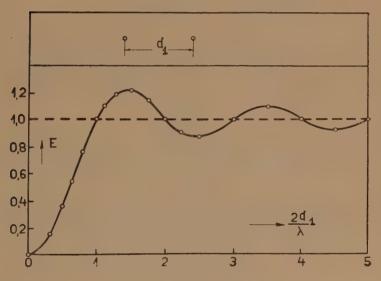
Graphs III., IV., and V. represent the intensity of sound in different directions as influenced by the enlargement of the baffle, and should be rotated about the vertical axis.

The last factor contains the influence of the distance d_1 between the sources. It gives, as a function of kd_1 , an oscillating curve, which, however, differs only slightly from 1 for values of kd_1 larger than $\pi/2$, or

$$d_1 = \frac{\lambda}{2}$$
.

We find again, as with the spherical model described first, that the energy emitted does not increase sensibly beyond a certain point, this saturation value of d_1 being equal to one-half of the wave-length. With our spherical model we found that the saturation value of half the sphere's circumference, being the shortest air-path between the sources, is equal to about one-half of the wave-length. Thus the assumption that the shortest air-path is the only preponderant factor in the problem is confirmed by comparison of the two cases considered, and we may expect that our spherical model is a fairly good representative of a flat baffle.

Graph VI.



APPENDIX.

In calculations of the kind carried out above, a table of the functions

$$F_n(ix) = \alpha_n + i\beta *$$

is of great use. As no such table has come to my know-ledge, I thought it well to give the values which I calculated for the above purpose in Table II. Rayleigh gave the values of F_0 to F_5 for $x=\frac{1}{2}$, and of F_0 to F_7 for x=1 and x=2. The last figure in the table is uncertain, a slide-rule of 25 cm. having been used.

TABLE II.

x	•5	0.8	1.3	1.5	2.0	2.2	2.5	3.0	3•5	4.0	4.5	5.0
a,	2.00	2.00	2.00	2.00	2.00	2.00	2.00	. 2.00	2.00	2.00	2.00	2.00
β_1	-3.50	-1.70	.240	·1 6 9	1.00	1.29	1.70	2.33	2.93	3.50	4.01	4.60
a ₈	•••	•••			-8.00	-5.40	-2.60	.33	2.10	3.25	4.04	4.60
β_3	•••	•••	***	5.00	4.00	- 4.44	-4.46	-3.78	-2 ·81	-1.82	84	•08
$a_{\scriptscriptstyle S}$	•••	***	***	***	***	***		4.30	-6.20	-7.80	-6.50	-4.31
β_5	•••				•••	•••		32.0	15.5	5.70	·22	-2.80
a ₇	***			•••	•••	***	~ ***	***	***	***	***	27.1
β_7	•••		•••	•••	•••			•••	•••	•••		8.00

Physical Laboratory of
Philips's Glowlamp Works Ltd.,
Eindhoven, Holland.
October 1928.

LXIII. A Dynamical Method for the Determination of Young's Modulus by Bending. By Kamta Prosad, M.Sc. (Cal.), B.A. (Cantab.)*.

Introduction.

THE purpose of this communication is to describe a method tor the determination of Young's modulus of materials in the shape of bars by dynamical bending.

Theory.

It has been shown by the late Lord Rayleigh † that for lateral vibration of bars the frequency is given by

$$\nu = \frac{k \cdot b}{\pi 2 L^2} \cdot m^2,$$

where

k = radius of gyration of the section about an axis perpendicular to the plane of bending.

b = velocity depending on the material of which the bar is made.

m = an abstract number depending on the mode of vibration.

L = length of the vibrating portion of the bar fixed at one end and free at the other.

^{*} Communicated by the Author.
† 'Theory of Sound,' 2nd ed. i. p. 273

If the section of the bar be rectangular and of thickness t in the plane of vibration, then $k^2 = \frac{1}{12}t^2$ and b is nothing else than the velocity of sound in the material of the bar, and is given by

$$b = \sqrt{\frac{q}{\rho}},$$

where q is the Young's modulus and ρ the density of the material of the bar.

We thus have

$$\nu = \frac{\sqrt{q} \cdot t}{\sqrt{\rho} \cdot \sqrt{12} \cdot 2\pi L^2} \cdot m^{\parallel}$$

or

$$v^2 = \frac{t^2 \cdot m^4}{48\pi^2 L^4} \cdot \frac{q}{\rho}$$

which can be put in the form $y = A \cdot x$, representing a straight line through the origin by putting

$$y = v^2,$$

$$x = \frac{1}{L^4},$$

and

$$A = \frac{t^2 \cdot m^4}{48\pi^2} \cdot \frac{q}{\rho}.$$

For the gravest mode of vibration *, m = 1.8751. Thus, by plotting ν^2 against $\frac{1}{L^4}$ a straight line would be obtained, and the tangent of the angle which this straight line makes with the axis of $\frac{1}{L^4}$ would be given by

$$\tan \theta = A = \frac{q}{\rho} \cdot \frac{t^2 \cdot m^4}{48\pi^2}.$$

As t and m are known, $\frac{q}{\rho}$ could be determined, and knowing

p for the bar, we can calculate q.

The author is not aware if this method has been previously used for the determination of Young's modulus. Possibly one reason for not making use of the above simple relation might have been the uncertain end-correction to be made to

the vibrating portion of the bar, as the effect of any attachment to it for the purpose of determining its frequency is to alter its mode of vibration.

End-correction.

The method adopted here for the purpose of making endcorrection is to assume that the effect of adding any load at the free end of the bar is to effectively lengthen it, and will be clear from what follows.

For, let the vibrating length of the bar be l_1 with the endoad on, and its frequency ν_1 . Then ν_1 is really the frequency of an unloaded bar of length $(l_1 + \delta)$, where δ is the endcorrection.

The frequency according to the relation given before for an unloaded bar must be given by

$$\nu_1 = \frac{t \cdot \sqrt{q}}{2\pi \sqrt{12} \cdot \sqrt{\rho}} \cdot \frac{1}{(l_1 + \delta)^2} \cdot m^2.$$

For another length l_2 of the bar, but with the same end-load,

$$\nu_2 = \frac{t \cdot \sqrt{q}}{2\pi \sqrt{12} \cdot \sqrt{\rho}} \frac{1}{(l_2 + \delta)^2} \cdot m^2$$

or

$$\frac{\nu_1}{\nu_2} = \frac{(l_2 + \delta)^2}{(l_1 + \delta)^2}.$$

As ν_1 and ν_2 are experimentally determined and l_1 and l_2 are read off the bar, we can determine δ from the quadratic equation above formed. The positive value of δ is then taken as the end-correction.

Experimental.

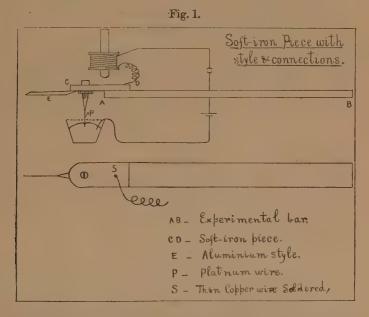
The important quantity to be determined in the formula is the frequency of the vibrating bar. For this purpose a smoked cylinder rotating at a known speed was used, and an aluminium style fixed to the free end of the bar kept vibrating electrically was allowed to record its vibrations on the drum. For each length of the bar at least two separate records were obtained from which the mean frequency was calculated.

It might be worth while recording here that for the electrical maintenance of the vibrations it was necessary to fasten a small thin sheet of soft iron by means of a little sealing-wax at the free end of the bar when non-magnetic

materials were used for experiment. With the aid of this device it was possible to work with all kinds of materials. The design of the style with the soft iron piece is shown in fig. 1. The vibrating bars were clamped firmly in a vice fixed to a strong table at any desired cm. mark made on them.

A Specimen Calculation and Procedure.

A curve is drawn between readings of the cm. divisions at which the bar is successively clamped, and the corresponding frequencies determined from the smoked drum.



From the curve so drawn, frequencies at some points of the clamp other than those actually used in the experiment are also read.

The length between the cm. mark at which the bar is clamped and some convenient division near the free end beyond which the soft iron piece is fixed is noted for each position of clamp of the bar. The end-correction then applies to this division taken as the zero. Thus, if beginning from the free-end as 0 cm. mark, the bar is clamped at 21 cm. mark and the iron piece extends up to, say, near 3 cm. mark, the length between the 21st cm. mark and the 4th cm. mark is taken, that is, l = (21-4) = 17 cm. The corrected length,

then, is $(17+\delta)$. For other points of clamp the same

procedure is followed.

For accurately determining δ_1 , four values of l and the corresponding ν are chosen well distributed over the $l-\nu$ curve. Two equations are then formed and δ determined from each independently. The mean of the two is then taken to be the end-correction.

As an example we will take the case of brass:—

The two equations for determining δ are:

$$\frac{70.50}{25.87} = \frac{(11+\delta)^2}{(4+\delta)^2} \dots \text{ (i.)} \quad \text{and} \quad \frac{49.50}{18.75} = \frac{(14+\delta)^2}{(6+\delta)^2} \dots \text{ (ii.)}.$$

From (i.) we get

or

$$\delta^2 - 0.14\delta - 45.04 = 0,$$

$$\delta = +6.78 \text{ or } -6.64.$$

From (ii.), similarly, we have

$$\delta^{2} + 2 \cdot 24\delta - 61 \cdot 56 = 0,$$
or
$$\delta = +6 \cdot 80 \text{ or } -9 \cdot 04.$$

Thus mean $\delta = 6.79$ cm.

Table I. gives the results of observations:—

TABLE I.

Obs.	ν.	$v^2 = y$.	l, in cm.	$L = (l + \delta),$ in cm.	$rac{1}{L^4} = x.$
1	15.90	252.81	17.0	23.79	3·12×10-6
2	18.75	351.56	14.0	20.79	5.35 ,,
3	25.87	669-26	- 11.0	17:79	9.98 ,,
4	37.25	1387.56	8.0	14.79	20.89 ,,
5	49.50	2450:25	6.0	12.79	37:37 "
6	70.50	4970.25	4.0	10.79	73.77 "
7	85.15	7250.52	3.0	9.79	108.86 ,,

The result is shown graphically in fig. 2, from which

Mean
$$\tan \theta = 6.67 \times 10^7$$
.

Thus
$$A = \tan \theta = \frac{q \cdot t^2 \cdot m^4}{\rho \cdot 48\pi^2} = 6.67 \times 10^7,$$
or
$$\frac{q}{\rho} = \frac{6.67 \times 10^7 \times 48\pi^2}{(0.1527)^2 \times (1.8751)^4}$$

$$= 1.096 \times 10^{11},$$
or
$$q = 1.096 \times 8.48 \times 10^{11} \text{ dynes/cm.}^2$$

$$= 9.29 \times 10^{11} \text{ dynes/cm.}^2$$

The results obtained with a few other materials by the method outlined in this paper are given in Table II. For the sake of comparison, the Young's moduli of the very same bars were also determined by the ordinary flexure method and are included in the table.

TABLE II.

No.	Material.	Density.	q—by Author's method, in dynes/cm. ²	q-by flexure method, in dynes/cm. ²	q—from Kaye & Laby's table of constants, in dynes/cm,2
1	Steel.	7.80	19·49×10 ¹¹	20·26×10 ¹¹	19·5-20·6×10 ¹¹
2	Brass.	8.48	9.29 ,,	9.79 ,,	9.7-10.2 ,
3	Glass.	2.54	7.56 "	6 ·3 8 "	6.5- 7.8 "
4	Ebonite.	1.255	0.091 "	0.24 "	×

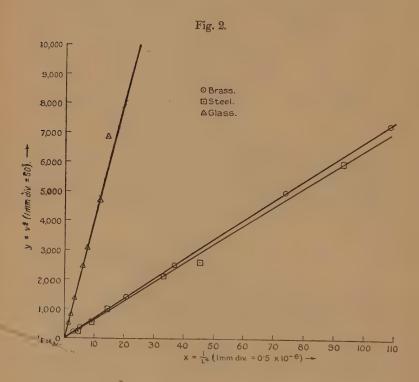
It would be noticed by reference to fig. 2 for ebonite that it is not straight over the whole course and does not pass through the origin. The explanation of this is that the soft iron piece was too heavy for the ebonite strip used, and that during the oscillations the ebonite got strained beyond the elastic limit. Over the portion of the curve convex to

the axis of $\frac{1}{L_4}$, the value of $\tan \theta$ continuously decreases,

giving smaller and smaller values to q.

Phil. Mag. S. 7. Vol. 7. No. 43. March 1929.

In conclusion, I have to thank Mr. K. Dayal, M.Sc., of the Physics Department, for kindly determining the values of q for the materials by the flexure method. I have also to



thank Mr. R. P. Gupta, M.Sc., for help in determining the frequencies of the vibrating bars.

[Added in proof.—It has not been possible to show the graph conveniently for ebonite in fig. 2 on the scales adopted for the other materials.—K. P.]

Physics Department, Science College, Patna, India. LXIV. The Energies of Dissociation of Cadmium and Zinc Molecules from an Interpretation of their Band Spectra. By J. G. WINANS, Ph.D., National Research Fellow*.

[Plate XIV. figs. 1 & 2.]

Introduction.

THE absorption spectrum of cadmium at pressures below 1400 mm. has been studied by Mohler and Moore (1). They have observed a group of bands near 3261, a continuous band extending from below 2288 to longer wavelengths and terminating in a series of flutings between λλ 2781 and 2653, and a band at 2212. Jablonski (2) observed a continuous band from 2112–2135 with a maximum of intensity at 2114. Further observations of these bands described below show very interesting changes with pressure which can be interpreted on the basis of a set of molecular energy levels.

APPARATUS.

The absorption cells, one 32 cm. and one 18.7 cm. long, were made of fused quartz with flat end-plates. Cadmium or zinc was distilled into the cell after evacuation and baking, and the cell was sealed off at a temperature near 600° C. It was then supported in a furnace arranged to prevent condensation of metal on the windows. A hydrogen discharge-tube similar to one described by Lambrey and Chalonge (3) served as a good source of continuous light. Spectra were photographed with sizes E 2 and E 31 Hilger quartz spectrographs.

ABSORPTION SPECTRUM OF CADMIUM.

The development of the band absorption in cadmium vapour as the pressure is increased is shown on Pl. XIV. (fig. 1, #1-16). At low pressures the 2288 (1¹S-2¹P) cadmium resonance line appears as a narrow absorption line, and then broadens symmetrically with pressure until it is over 60 Å. wide. It overlaps the 2212 band at pressures greater than 60 mm., but does not broaden beyond this band at 186 mm., although the long-wave limit extends as far as 2530 Å. At pressures over 130 mm. a set of broad and diffuse flutings are seen on the long wave-length side

^{*} Communicated by the Author.

of this band. These flutings extend from $\lambda\lambda\,2650$ to 2780 and converge toward the violet. Their wave-lengths and frequencies are given by Mohler and Moore (1). The bands at 2212 and 2114 appear at 7 mm. The 2212 band broadens a little toward longer wave-lengths, and the 2114 band broadens slightly in both directions with increasing pressure.

THE SPECTRUM OF THE ELECTRODELESS DISCHARGE IN CADMIUM.

· Cadmium contained in a sealed quartz tube was heated by a Bunsen flame, and a discharge was produced in the tube through a single external electrode attached to a low-voltage Tesla coil.

In fig. 1 (Pl. XIV., #17) is seen the spectrum of this discharge. It showed principally the lines of the cadmium arc spectrum, but the higher series lines near the convergent limit of the triplet S and D series were more strongly developed than in the spectrum of the cadmium arc. The band at 2114 appears strongly, but the 2212 band failed to appear. These two bands at 2212 and 2114 have nearly equal intensities in absorption, but only the 2114 band appears in the electrodeless discharge. This is seen on comparing #17 with #1-16 in fig. 1 (Pl. XIV.).

A continuous spectrum extends from 4800 to 2240 with maxima of intensity at 4400, 3300, 2980, and 2288. There is a decrease in intensity at 3260, a faint maximum at 2980, then a gradual increase in intensity to 2288, and rapid decrease to zero at wave-lengths less than 2288. The last

observable trace is at 2220.

Two spark resonance lines, 2133.4 and 2265.0, and faint traces of a few other spark lines were observed in the spectrum of the electrodeless discharge.

THE ABSORPTION SPECTRUM OF ZINC.

The absorption spectrum of zinc is similar to that of cadmium. At pressures over 730 mm., Mohler and Moore (1) observed a group of bands near 3000, a set of flutings from 2636-2551, and continuous absorption below 2550. With the arrangement of apparatus as previously described (4), other absorption bands at 2064 and 1997-2006 are observed. These bands are shown on Pl. XIV. (fig. 2, #1-3). The zinc absorption line at 2139 (1¹S-2¹P) broadens into a band with increasing pressure, exactly as the cadmium resonance line 2288 shown on Pl. XIV. (fig. 1, #1-16).

The short-wave limit of the zinc band comes at 2064. The flutings observed by Mohler and Moore were not found in this work—probably due to insufficient vapour-pressure.

DISCUSSION.

The Band Spectrum of Cadmium.—The observations on the cadmium band spectrum can be briefly stated as follows. The absorption line 2288 broadens into a band with increasing pressure. It reaches a definite limit at 2207 on the violet side, and on the red side extends to 2800, terminating in a series of flutings. These flutings converge towards the violet. Two absorption bands appear at 2212 and 2114, but only the 2114 band is observed in emission in the electrodeless discharge. In the electrodeless discharge the cadmium line 2288 is surrounded by a continuous spectrum which decreases in intensity rapidly at wave-lengths below 2288, and gradually for wave-lengths above 2288.

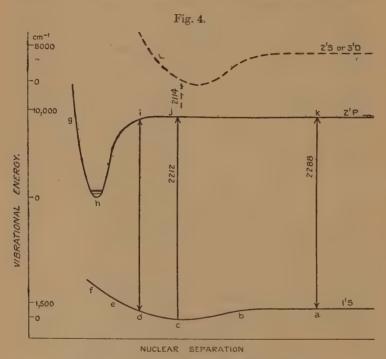
It is natural to seek for an interpretation of these observations in terms of absorption and emission by cadmium molecules. A clear way to picture the absorption and emission processes in a molecule is to draw the curves of vibrational energy as nuclear separation for the different electronic states of the molecule. By a series of arguments, which will be presented below, one can show that two of the electronic states in the cadmium molecule are probably represented by an arrangement of curves like those shown

in fig. 4.

Granting for the moment that the relative shapes and positions of the curves are correct, we can see how the experimental observations can be explained. According to a principle first given by Franck (6) and extended by Condon (6), only those transitions will occur for which the position and momentum of the nucleii in the initial state equal the position and momentum in the final state. Further, since a vibrating dipole spends most of its time near the ends of its vibrations, transitions are most likely to occur when the nuclei are at either their maximum or minimum displacements. This means that in fig. 4 the most probable transitions are represented by vertical lines from the ends of the vibration levels on one potential energy curve to the ends of the vibration levels in the curve for another electronic state.

In fig. 4, the 2212 band is represented by an absorption *cj* from the non-vibrating state of the normal molecule to the flat part of the potential energy curve of the excited state.

This means that after absorption of 2212 by a Cd₂ molecule the two cadmium atoms will be free to drift apart, one of them being in the 2¹P atomic state. As soon as the pressure is high enough for the formation of a sufficient number of molecules to show absorption, the 2212 band should appear, and with increasing pressure and temperature should broaden only towards the red. It represents the shortest wave-length absorption possible with the curves as shown in fig. 4. This agrees with the development of the 2212 band with pressure



Potential-energy curves for the cadmium molecule.

as shown in Pl. XIV. (fig. 1, #1-16). There is no atomic line of cadmium at 2212. This band must be attributed to molecular absorption. Also, this band being represented by an absorption from a stable molecular state into an unstable one should appear in absorption but not in emission, and this is exactly that which was observed. Compare numbers 17 and 1-16 in fig. 1 (Pl. XIV.).

The broadening of the 2288 line into a band, with pressure, is represented by absorptions from quasi-molecules,

as discussed by Born and Franck (7). When two atoms collide, their mutual potential energy follows the potential energy curve for the molecule. If either atom absorbs light during collision, the wave-length absorbed will be determined by the molecular energy-level curves. slowly-moving atoms in collision, following the curve abcdef in fig. 4, have their maximum velocity at c on approach and departure, and come instantaneously to rest near d. The probability of absorption of light is greatest near d and b, and least near c, due to the relative amount of time spent at these places during a collision. Absorption from the region de gives a wave-length longer than the atomic line, and absorption from dc and cb gives a wave-length shorter than the atomic line. With increasing pressure and energy of the colliding atoms, the absorption will extend to longer wave-lengths corresponding to absorption from the region def, and to shorter wave-lengths as far as the limit set by the 2212 band (absorption cj). The 2288 cadmium line should then broaden with pressure into a band, limited on the shortwave side by the 2212 band, but not so limited on the long-wave side. This asymmetrical broadening is exactly that shown in fig. 1, #1-16 (Pl. XIV.).

The electrodeless discharge shows the 2288 line in emission surrounded by a continuous spectrum. This is represented in fig. 4 by the excitation of quasi-molecules from the region cde through electron impacts. A stable excited molecule is formed which later emits a continuous radiation corresponding to the energy received. The probability of excitation is again greatest at d, and the energy received corresponds to wave-lengths near 2288. The main features of the absorption band should then appear in the emission band, but without the emission of the 2212 band, since the initial state for this band is unstable. The continuous spectrum surrounding 2288, shown in fig. 1, #17 (Pl. XIV.), has the features expected from fig. 4, and the 2212 band failed to appear as also expected from fig. 4. The faint maximum at 2980 may be due to emission by excited molecules which have lost their vibrational energy through

So far we see that the observations shown in fig. 1 (Pl. XIV.) on the absorption bands 2212 and 2207-3000 in cadmium can be explained by a set of potential-energy curves as shown in fig. 4. Also, these curves explain the continuous spectrum surrounding the 2288 line and the non-appearance of the 2212 band in the electrodeless discharge.

collisions of the second kind.

This success in explaining observations is evidence that the curves in general characteristics are correct, but one naturally wishes also for a more general justification on the basis of band-spectrum constants. The band-spectrum constants are not known; but from analogy with other molecules it can be shown that these curves are to be

expected for the cadmium molecule.

Zinc, cadmium, and mercury vapours are non-atomic at low pressures; at pressures over 10 mm., molecules are formed which give rise to band absorption. By measuring the decrease in intensity with superheating of the mercury band at 2540, Franck and Grotrian (8) and Koernicke (9) computed the energy of dissociation of Hg₂ to be about 0.06 volt. An energy of dissociation of this order of magnitude is to be expected for cadmium and zinc. If Cd₂ has a low heat of dissociation, it is seen through an empirical relation of Bates and Andrews (10),

$$\frac{D}{(\omega^0)^2 \mu} = \text{a constant for all homopolar molecules to}$$
 within 30 per cent.,
$$D = \text{energy of dissociation,}$$
 (1)

that $\omega^{0''}$ is small. A small value of $\omega^{0''}$ means a wide shallow potential-energy curve for the normal state. The potential-energy equation is $^{(11)}$

$$v = u_2(r - r_0)^2 + u_3(r - r_0)^3 + \dots, u_2 = [0.95641 - 3][(\omega^0)^2 \mu].$$
 (2)

The quantity in brackets is the logarithm of the coefficient. v and ω^0 are in cm⁻¹, $r-r_0$ in Angström units,

$$\frac{1.65}{\mu} = \frac{1}{M_1} + \frac{1}{M_2},$$

where M_1 and M_2 are the atomic weights of the atoms. The parabolic approximation to equation (2) given by the first term can be obtained when only ω^0 is known.

The continuous band surrounding 2288 extends over a range corresponding to an energy greater than that which we should expect for the energy of dissociation of the normal molecule. It is then necessary to attribute this wide extent to the range of vibration levels in the excited state, and say that the energy of binding of the excited molecules is greater than that of the normal molecules. This is almost certain for such molecules as Zn_2 , Cd_2 , and Hg_2 , and means in general, though not always, that $\omega^{0'} > \omega^{0''}$ (from 1), the

potential energy curve for the excited state, is narrower than that for the normal state (from 2), and the minima come at $r_0' < r_0''$. This last comes from another empirical rule,

or
$$\omega^{0''} \mathbf{I}_0^{\ ''} = \omega^{0'} \mathbf{I}_0^{\ '} \ \omega^{0''} r_0^{\ ''2} = \omega^{0'} r_0^{\ '2} \$$
 (3)

to within 20 per cent. for the different states in one molecule. When the atoms of a molecule are separated, the curves of potential energy against nuclear separation become the energy levels of the atoms. The limit of the curve abcdef for the normal cadmium molecule is the normal state of the cadmium atom 1¹S. The limit of the curve ghijk for the excited state is taken as the 2¹P cadmium atomic state. This is indicated by the development of the absorption band out of the line $1^1S - 2^1P \ \lambda \ 2288$, as shown in fig. 1, $\sharp \ 1-16$ (Pl. XIV.). Also, the $1^1S - 2^1P$ is the only cadmium atomic transition in the region of the band from 2207-2800.

In this way, from the assumption of a low heat of dissociation for Cd₂ in analogy with mercury, and associating the width of the band from 2207-2800 with the range of vibration states in the excited molecule, we are led to a probable picture of the energy levels of Cd₂ as shown in fig. 4. These levels are in themselves consistent, and explain the experimental facts as described above. The relative positions shown in fig. 4 are the only positions which will explain the observations.

In fig. 4 the energy of dissociation of Cd₂ is given by the difference in energy between points a and b on curve abcdef, and this difference is given by the difference in energy between the 2288 line and the short-wave limit of the 2212 band. This energy difference is 1600 cm.⁻¹, 0.200 volt, or 4.7 kg. cal.

The Band Spectrum of Zinc.—The general features of the zine absorption bands, shown in fig. 2, #1-3 (Pl. XIV.), are the same as those for the cadmium bands. Assuming the same kind of curves as those of fig. 4, the energy of dissociation of Zn₂ is 2000 cm.⁻¹, 0.246 volt, or 5.7 kg. cal.

Energies of Dissociation of Zn_2 , Cd_2 , and Hg_2 .—It is interesting to compare the energies of dissociation of zinc, cadmium, and mercury molecules, as obtained here and by Koernicke, with the heats of fusion. The energy absorbed in melting a metal into freely-moving atoms might be expected to be proportional to the energies of dissociation of the molecules. This is found to be true for zinc, cadmium, and mercury, as shown in Table I.

Absorption Coefficients for Singlet and Triplet Systems .-One might wonder why no band appears near the cadmium line 3261 (11S-23P1) due to molecular states which converge toward the three triplet P-levels. By comparison of the pressures at which $2288 (1^{1}S-2^{1}P)$ and $3261 (1^{1}S-2^{1}P)$ first appear in absorption, one can estimate the relative absorption coefficients of cadmium vapour for these lines. In this way it is found that the coefficient for 2288 is 7000 times that for 3261. In fig. 1, #16 (Pl. XIV.) the band absorption in the region of 2288 is fully developed, while the 3261 line has just appeared as a narrow line, and there is no sign of bands in the region of 3261. Since the transitions between levels of the singlet system are so much more probable than those between singlet and triplet systems, we need to consider only the singlet terms in our discussion of the absorption bands near 2288 shown in fig. 1 (Pl. XIV.).

Table I.

Comparison of Heats of Fusion and Energies of Dissociation.

Metal.	$_{ m cal./grm.}^{ m H,}$	Atomic weight.	H, cal./mole.	$H \times \frac{4}{3} \times 10^{-4}$.	D, volts.
Zn	2 8	65	1800	0.24	0.25
Cd	13.7	112	1500	0.20	0.20
Hg	2 ·8	200	560	0.07	0.06

The Cadmium Band at 2114.—This band was observed by Jablonski ⁽²⁾, who found it in absorption and in fluorescence. It was also observed in the electrodeless discharge by Kapuscinski, and by de Groot in the positive column discharge through a mixture of cadmium and argon. It was observed in this work in absorption, and in emission in the electrodeless discharge. Jablonski ⁽²⁾ also found that absorption by cadmium vapour of light in the region of this band resulted in the emission of the triplet 2^3P_2 , $_1$, $_0-2^3S$ in fluorescence.

This band, which is observed both in emission and absorption, should be represented by a transition between stable molecular states. Its appearance in absorption at about the same pressure as 2212 (see fig. 1, #1-16 (Pl. XIV.)) indicates that it is also due to an absorption by normal Cd₂ molecules. Since the singlet-triplet absorptions are very much less probable than the singlet-singlet absorptions, it is natural-

to associate the 2114 band with a transition from the normal molecule to a molecular state whose vibration levels converge to some singlet atomic level above 2¹P. The next singlet atomic levels are 2¹S and 3¹D.

Representing a molecular state by the dotted curve shown in fig. 4, one can explain the broadening of the 2114 band with pressure, the difference of wave-length of the maximum in absorption and emission observed by Jablonski ⁽²⁾, the fluorescence observations of Jablonski ², and the appearance of this band in absorption at nearly the same pressure as 2212. The sharp triplet in fluorescence is assumed to result from collisions of the second kind between excited cadmium molecules and normal cadmium atoms.

This hypothesis for the cadmium band 2114 applies also to the corresponding zinc absorption band at 2002 shown in

fig. 2 (Pl. XIV.).

Flutings in the Band Spectrum of Cadmium.—Mohler and Moore (1) observed a series of flutings between 2781 and 2653 in absorption, and Kapuscinski and Van der Lingen found a similar set of flutings between 3000 and 2650 in fluorescence in cadmium vapour. Kapuscinski also found that cadmium vapour re-emitted in fluorescence several lines of wave-length within the limits of the fluorescence band 3000-2288. Walter and Barratt have recently stated that these flutings may be due to impurities such as CdO. If this is true, it must be assumed that in the region between 2800 and 2300 the bands of the impurities are so closely spaced that practically any frequency in this region can be absorbed and later emitted, in accordance with the observations of Kapuscinski.

These observations, however, can be explained on the basis of fig. 4, and the flutings attributed to Cd2 molecules. Two colliding cadmium atoms which absorb energy and form a stable excited molecule will from the Franck-Condon principle, favour those transitions which can be represented by vertical lines from the region def to the ends of the vibration levels in the excited state. When one curve is much steeper than the other, as those in fig. 2, this may result in rises and falls of intensity in the continuous absorption region. This would appear as a series of flutings on the long wave-length edge of the absorption band which grows out of 2288. Two colliding atoms which absorb light in the region 2800-3000 will form an excited molecule which can later emit the energy received as a fluting of the same wave-length. This is a possible explanation of the observations.

SUMMARY.

1. New observations on the absorption spectrum in cadmium vapour show the development of the atomic line 2288 into a band from 2207-2800 with increasing pressure. The absorption bands at 2212 and 2114 were also observed.

2. The electrodeless discharge in cadmium vapour shows a continuous spectrum with maxima at 2288, 2980, 3000, and 4400. The band at 2114 appears, but the 2212 band is

noticeably absent.

- 3. The cadmium band from 2207 to 2800 is attributed to absorption of light by two colliding atoms, the end product being a stable excited molecule. This view explains the development of the absorption band with pressure, the appearance of the 2212 band in absorption but not in emission, the continuous spectrum in the electrodeless discharge between 2220 and 3000, and yields a value of 0.200 volt for the energy of dissociation of Cd₂.
- 4. Applying the same explanation to the absorption spectrum of zinc gives an energy of dissociation of 0.246

volt for Zn2.

- 5. The energies of dissociation of zinc, cadmium, and mercury molecules are found to be proportional to the atomic heats of fusion.
- 6. The 2114 band of cadmium and the corresponding band at 2002 in zinc can be represented by an absorption to a stable molecular state whose vibration levels converge to 21S or 31D.
- 7. A possible explanation is given for the flutings between 3000 and 2650 observed in absorption and fluorescence.

In conclusion, I wish to express my thanks to Professor K. T. Compton for the privilege of working in the Palmer Laboratory, and to Professors E. U. Condon and F. W. Loomis for helpful discussions on the subject of this paper.

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(1) F. L. Mohler and H. R. Moore, J. O. S. A. xv. p. 74 (1927).

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(7) M. Born and J. Franck, Zeit. f. Phys. xxxi. p. 416 (1925).
(8) Franck and Grotrian, Zeit. f. Tech. Phys. iii. p. 194 (1922).

(9) Koernicke, Zeit. f. Phys. xxxiii. p. 219 (1925).

(10) J. R. Bates and D. H. Andrews, Proc. Nat. Acad. Sci. xiv. p. 124 (1928). The form given here is obtained by eliminating the K between the two equations given by Bates and Andrews. When D is in volts, ω⁰ in cm.⁻¹, and μ in units of 10⁻²⁴ grm., the value of the constant is 2.9 × 10⁻⁷.

(11) E. U. Condon, loc. cit. p. 1194.

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LXV. Flutings in the Absorption Spectrum of a Mixture of Mercury and Cadmium Vapours. By J. G. WINANS, Ph.D., National Research Fellow *.

[Plate XIV. fig. 3.]

DURING the experiments described in the preceding paper, one absorption cell was prepared with a

mixture of cadmium and mercury.

In the absorption spectrum of this mixture of cadmium and mercury the 2288 line of cadmium appears and broadens with pressure into a band in the same way as in pure cadmium, and a new set of flutings is observed on the long wave-length edge of the band. They are shown on Pl. XIV. (fig. 3, \$\pm\$3 and 4). These flutings are diffuse, like those in pure cadmium, and are difficult to measure accurately. Their wave-lengths and frequencies are given in Table I. Wave-lengths were measured at intensity maxima.

The partial pressures of mercury and cadmium at which these flutings appear may be obtained approximately by comparing the widths of the mercury and cadmium bands in the mixture with a series of absorption spectra of pure mercury and cadmium taken at different pressures. In this way it is found that the cell contained mercury at 100 mm.

and cadmium at 60 mm. pressure.

These flutings are similar in character to flutings observed in mercury, cadmium, and zine by Rayleigh (1) and Mohler and Moore (2). Walter and Barrett (3) have recently mentioned having evidence that some of these flutings are due to impurities. This may also be true of these flutings in the mixture of mercury and cadmium, although they were not observed in mercury alone, or cadmium alone, at pressures up to 180 mm. If they were due to Hg₂, Cd₂, or

^{*} Communicated by the Author.

TABLE I.

Flutings in the Absorption Spectrum of a Mixture of
Cadmium and Mercury.

Wave-length.	Wave-number.	$\Delta \nu$.
2488·6	40171	140
2480.0	40310	122
2472.5	40433	
2465.6	40545	112
2459•4	40648	_ 103
2453.6	40744	96
2448.5	40829	85
2443.6	40911	82
		80
2438.8	40991	79
2434.1	41070	69
2430.0	41139	67
2426.1	41206	
2422.3	41270	66
2418.7	41331	61
		77
2414.2	41408	57
2410.9	41465	69
2406.8	41536	30

an impurity in either, they would be expected to appear in mercury or cadmium alone. They are therefore most likely due to HgCd molecules.

References.

(1) Rayleigh, Proc. Roy. Soc. Lond. cxiv. p. 620 (1927); cxvi. p. 702 (1927).

(2) F. L. Mohler and H. R. Moore, J. O. S. A. xv. p. 74 (1927).
(3) J. M. Walter and S. Barratt, 'Nature,' cxxii. p. 684 (Nov. 3, 1928), p. 748 (Nov. 10, 1928).

Palmer Physical Laboratory, Princeton, New Jersey, U.S.A. August 30, 1928. LXVI. The Measurement of Conductivities by means of Oscillating Circuits. By S. D. Gehman and B. B. Weatherby, University of Pennsylvania*.

In a recent issue of the Philosophical Magazine †, Burton and Pitt report on certain experiments in which they claim to have measured the conductivities of various solutions and organic liquids. The paper is entitled "A New Method of Conductivity Measurement by means of an Oscillating Valve Circuit."

We wish to point out that the changes in distributed capacity caused by inserting the glass tube containing the liquids worked with into the coils of the oscillating system described by Burton and Pitt are sufficient to produce serious frequency changes in the circuit—changes which would affect the interpretation of the results recorded. It is surprising that the losses on which the authors must be depending to furnish an indication of the conductivity do not continue to increase the effects as the conductivities of the solutions are increased. The changes in instrumental deflexion with change in concentration of the KCl solutions are certainly not what one would expect on the assumption that the relation between the conductivity and concentration within the limits used is a linear one.

It is our opinion that the effects reported by Burton and Pitt are to be ascribed to the dielectric constants of the liquid used. We tabulate below the deflexions they publish for organic liquids, and in a parallel column the accepted values for the dielectric constants of these liquids.

Liquid.	Deflexion.	Dielectric constant.
Hexane	6.8	1.9
Pentane	8.8	1.8
$\left\{egin{array}{lll} \mathbf{X}\mathbf{y} \mathbf{lene} & \dots & \dots & \dots & \dots \\ \mathbf{X}\mathbf{y} \mathbf{lol} & \dots & \dots & \dots & \dots \end{array} \right.$	$\left\{ egin{array}{l} 15.3 \ 11.8 \end{array} ight.$	2.5
{ Toluene	$\left\{ \begin{matrix} 14\cdot 0 \\ 15\cdot 1 \end{matrix} \right.$	2:3
Carbon tetrachloride	17.2	2:2
Benzol	18·0	2 ·3
Amyl alcohol	23.7	16.0
Ether	32.3	4.3
Chloroform	42.5	5.2
Butyl alcohol	71.4	19.2
Ethyl alcohol	77.2	25 ·8

^{*} Communicated by Prof. C. B. Bazzoni, Ph.D.

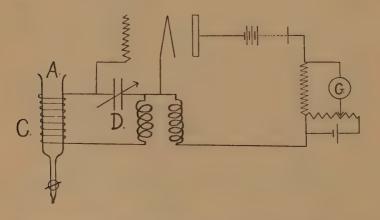
† Phil. Mag. v. p. 939 (1928).

The only marked exception to the ranking of deflexions in the order of the dielectric constants is in the case of amylalcohol.

In order to check this idea we built up a circuit duplicating that of Burton and Pitt in all essential features, and observed the deflexions obtained with acetone and glycerine. These were selected because their conductivities as compared by us in a usual manner were in the reverse order of their dielectric constants. It is to be observed that the deflexions are in the order of the dielectric constants and not in the order of the conductivities.

Liquid.	Comparative conductivity.	Dielectric constant.	Deflexion.
Acetone	5.6	21.5	44 cm.
Glycerine	1.0	56.2	66 ,,

We pushed the matter a little further, using the circuit shown in the figure.



The oscillation frequency was about 600,000 cycles per second. The circuit was quite sensitive enough for the purpose at hand. The liquids were introduced into the glass

vessel A, around which was wrapped a coil, C.

By using this arrangement any effects due to an alteration of the coupling between the plate and grid circuits and any losses in the plate circuit due to the introduction of the liquid were eliminated. This was found advisable because in the Burton and Pitt apparatus the deflexion depended upon how far the liquid was lowered into the plate coil.

The deflexions obtained with this arrangement duplicated satisfactorily those which we secured with the more com-

plicated circuit used by Burton and Pitt.

With this apparatus, deflexions were secured for solutions of KCl of various concentrations, and a curve was plotted. A condenser with paraffined plates was then shunted across D. This condenser was filled successively with the same solutions that had been used in the coil. The deflexions obtained were identical in nature with those secured when the coil was used.

We consequently feel safe in ascribing the effects obtained by Burton and Pitt primarily to the dielectric constants of the liquids used.

LXVII. The Motion of a Particle on a Rough Sphere, including the Case of a Rotating Sphere. By A. F. STEVENSON*.

In this paper we investigate the motion of a heavy particle on the interior of a rough sphere, which may be either fixed or may rotate with constant angular velocity about a vertical diameter. The case where the sphere is smooth—the spherical pendulum—is of course well known, and, in this case, the motion of the particle in space is unaffected by the rotation of the sphere (even if the axis of rotation is not vertical and the angular velocity not constant). The case where there is friction, however, does not appear to have received much attention †.

More positive conclusions can be drawn if a frictional force proportional to the velocity (or relative velocity) be assumed, rather than the more usual laws of friction, since the equations are thereby simplified and singularities avoided. This would correspond more nearly to the equivalent problem of the general motion of a single pendulum subject to air resistances, although, in this case, a resistance proportional to the square of the velocity would be nearer the truth. The case where the friction is of the usual type is also briefly considered.

* Communicated by G. I. Taylor, F.R.S.

[†] It is only in a few simple types of surfaces, or in artificially-constructed cases, that the motion on a rough surface can be solved by quadratures. Cf. N. Basu, Phil. Mag. (6) p. 35 (1918).

Phil. Mag. S. 7. Vol. 7. No. 43. March 1929.

Fixed Sphere.

1. Let a denote the radius of the sphere, θ the angle which the radius to the particle makes with the downward drawn vertical, ϕ the azimuth, and let the resistance to motion per unit mass be μ times the velocity. The equations of motion then are

$$\ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta = -\frac{g}{a} \sin \theta - \mu \dot{\theta}, \quad . \quad . \quad (1)$$

$$\frac{1}{\sin\theta} \frac{d}{dt} (\dot{\phi} \sin^2\theta) = -\mu \dot{\phi} \sin\theta . \qquad (2)$$

(2) may be written

$$\frac{dh}{dt} = -\mu h, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $h = \dot{\phi} \sin^2 \theta$. Multiplying (1) by $\dot{\theta}$, (2) by $\dot{\phi} \sin \theta$, and adding, we get

$$\frac{d\mathbf{E}}{dt} = -\mu(\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) = -2\mu \left(\mathbf{E} + \frac{g}{a} \cos \theta\right), \quad (4)$$

where

$$\mathbf{E} = \frac{1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) - \frac{g}{a} \cos \theta.$$

(3) and (4) give the rates of dissipation of angular momentum and energy, respectively; we see that E and h continually decrease. Since the dissipative forces are proportional to the velocity, these equations could also be obtained by using Rayleigh's "dissipation function"*.

2. From (3) we have

$$h = h_0 e^{-\mu t}, \qquad (5)$$

where h_0 is the initial angular momentum (we exclude the case $h_0=0$, which is that of the simple pendulum); hence the particle sinks asymptotically to the lowest position, as is evident a priori. To examine the motion more closely in the neighbourhood of $\theta=0$, substitute for $\dot{\phi}$ from (5) in (1), and retain only the lowest powers of θ . We thus obtain

$$\ddot{\theta} + \mu \dot{\theta} + \frac{g}{a}\theta = \frac{h_0^2 e^{-2\mu t}}{\theta^3}. \qquad (6)$$

Putting $\theta = e^{-\mu t/2} \psi$, this becomes

$$\ddot{\psi} = \frac{h_0^2}{\Psi^3} + \left(\frac{\mu^2}{4} - \frac{g}{a}\right)\Psi.$$

* Whittaker, 'Analytical Dynamics,' p. 230.

There are three types of solution for ψ :

(i.) If
$$\mu^2 - 4\frac{g}{a} > 0 (= \mu_1^2, \text{ say}), \quad \psi = A + Be^{\mu_1 t} + Ce^{-\mu_1 t}$$
.

(ii.) If
$$\mu^2 - 4\frac{g}{a} < 0 (= -\mu_2^2, \text{ say}), \ \psi = A' + B' \sin(\mu_2 t + C')$$

(iii.) If
$$\mu^2 - 4\frac{g}{a} = 0$$
, $\psi = A'' + (B''t + C'')^2$.

Here A, B, etc, denote certain constants depending on initial conditions. The general solution of (6) is thus obtained. We easily deduce that if $\mu^2 \ge 4\frac{g}{a}$, then $\dot{\theta}$ vanishes

at most once for finite values of t; but if $\mu^2 < 4\frac{g}{a}$, then

 θ vanishes, in general, for an infinity of values of t, so that a rising and falling of the particle in the sphere persists (these results are analogous to those for the small damped oscillations of a simple pendulum). Thus in the former case the particle spirals into the bottom of the sphere, and there is no periodic motion; but in the latter (small friction) the motion is of a quasi-periodic type, similar to that of the ordinary spherical pendulum. In the latter case there is, in particular, no tendency for the path of the particle to become circular, as might perhaps be supposed at first; the limiting form of the motion is a small ellipse, as may be shown by taking Cartesian instead of polar coordinates, just as in the ordinary spherical pendulum *, the size of the ellipse decreasing asymptotically to zero.

Moreover, when θ is small, (5) gives

$$\dot{\phi} = \frac{h_0 e^{-\mu t}}{\theta^2} = \frac{h_0}{\Psi^2},$$

from which the value of ϕ is easily obtained. We thus find that if $\mu^2 \ge 4\frac{g}{a}$, $\dot{\phi} \to 0$ and ϕ tends to a finite value, as $t \to \infty$; but if $\mu^2 < 4\frac{g}{a}$, $\dot{\phi}$ does not, in general, tend to any definite limit, and $\phi \to \infty$.

^{*} Reference may be made to any text-book where the spherical pendulum is treated in detail, e.g., Appell, 'Mécanique rationnelle,' vol. i. pp. 513 et seq.

3. So far no restriction has been placed on μ . If, however, we suppose μ small, then the particle may be considered to be moving approximately as in the ordinary spherical pendulum, and we can calculate the small corrections to be made. In this case, for values of t not too large, we may integrate (4) approximately in the form:

$$E = E_0(1 - 2\mu t) - \frac{2\mu g}{a} \int_0^{\pi} \cos\theta \, dt, \quad . \quad . \quad (7)$$

where E_0 is the value of E at t=0, and the integral on the right is calculated on the hypothesis of frictionless motion with the given initial conditions. In the spherical pendulum, $\cos \theta$ can be expressed as an elliptic function of the time in a form * which enables us to evaluate immediately the integral in terms of elliptic functions, if necessary.

Equations (7) and (5), which to the same order of

approximation may be written

$$h = h_0(1-\mu t), \dots (8)$$

now constitute two first integrals of the equations.

Suppose that at t=0 the path of the particle is tangent to the circle $\theta = \alpha$, having there a velocity which, in the absence of friction, would make the path also tangent to $\theta = \beta (\beta < \alpha)$ —there is no loss of generality in this—and let the values of $\dot{\phi}$ at $\theta = \alpha$, β be Ω_{α} , Ω_{β} respectively (these can, of course, be expressed in terms of α , β). Let us suppose that in the actual motion the next maximum of θ is given by $\theta = \alpha + \delta \alpha$. Then from (7), and using (8) to eliminate $\dot{\phi}$, we have for the value of E at $\theta = \alpha + \delta \alpha$

$$\begin{aligned} \mathbf{E}_{0}(1-2\mu\mathbf{T}) - \frac{2\mu g}{a} \int_{0}^{\mathbf{T}} \cos\theta \, dt \\ &= \frac{1}{2} \frac{h_{0}^{2} (1-\mu\mathbf{T})^{2}}{\sin^{2}(\alpha+\delta\alpha)} - \frac{g}{a} \cos(\alpha+\delta\alpha), \\ \mathbf{E}_{0} &= \frac{1}{2} \frac{h_{0}^{2}}{\sin^{2}\alpha} - \frac{g}{a} \cos\alpha; \end{aligned}$$

where

$$h_0 = \Omega_a \sin^2 \alpha;$$

and T denotes the "period"—i. e., the time between successive minima for θ —in the frictionless case. Retaining only first powers of μ and $\delta \alpha$, we obtain

^{*} Whittaker, l. c. p. 105.

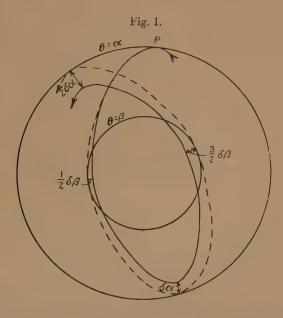
$$\delta \alpha = -\frac{2\mu(I - T\cos\alpha)}{\sin\alpha \left(1 - \frac{a}{g}\Omega_{\mu}^{2}\cos\alpha\right)}, \qquad (9)$$

where

$$\mathbf{I} = \int_0^{\mathbf{T}} \cos \theta \, dt.$$

Similarly, for $\delta\beta$, the increase in θ between successive minima for θ , we have

$$\delta\beta = -\frac{2\mu(T\cos\beta - I)}{\sin\beta \left(\frac{\alpha}{g}\Omega_{\beta}^{2}\cos\beta - I\right)}, \quad . \quad . \quad (10)$$



the value of the integral being the same owing to symmetry in the frictionless case. Since both numerator and denominator are positive in the expressions on the right of (9) and (10), we see that $\delta \alpha$, $\delta \beta$ are both negative, as we would expect.

The increase in θ at the first minimum after projection is evidently $\frac{1}{2}\delta\beta$, to the same order, and that at the second minimum $\frac{3}{2}\delta\beta$. The character of the path of the particle, projected on a horizontal plane, is shown in fig. 1 (for the case where the motion is entirely in the lower hemisphere). The particle is supposed projected from the point P

on the upper circle $(\theta = \alpha)$, and the dotted curve represents the horizontal projection of the path in the frictionless case, the initial velocity being the same. The distances marked $\delta \alpha$, etc., correspond to the corrections given by (9) and (10). The actual horizontal distances are of course $\alpha \cos \alpha \delta \alpha$, etc.

The integral I occurring in (9) and (10) may be expressed in terms of the period T by means of elliptic functions as in (7), or both I and T as elliptic integrals involving initial conditions. Thus we may consider the particle as moving between two horizontal circles on the sphere, these circles at the same time gradually sinking.

4. If $\alpha = \beta$, (9) and (10) fail, for then both numerator and denominator vanish. In this case we can use (1) directly. We now have

$$h_0^2 = \frac{g \sin^4 \alpha}{a \cos \alpha},$$

whence, substituting for $\dot{\phi}$ from (5) in (1),

$$\dot{\theta} - \frac{g \sin^4 \alpha \cos \theta}{a \cos \alpha \sin^3 \theta} e^{-2\mu t} = -\frac{g}{a} \sin \theta - \mu \dot{\theta}.$$

For an interval of time not too large we can write $\theta = \alpha + \xi$, and retain only the first powers of ξ , $\dot{\xi}$, and μ . We thus obtain

$$\ddot{\xi} + \frac{g}{a} \xi \sec \alpha \left(\sin^2 \alpha + 4 \cos^2 \alpha \right) = -\frac{2\mu g}{a} t \sin \alpha,$$

whence the solution, with $\xi = \dot{\xi} = 0$ when t = 0,

$$\xi = \frac{\mu \sin 2\alpha}{\sin^2 \alpha + 4 \cos^2 \alpha} \left(\frac{1}{p} \sin pt - t \right),$$

where

$$p^2 = \frac{g}{a} \sec \alpha \left(\sin^2 \alpha + 4 \cos^2 \alpha \right).$$

Thus, when next the path is horizontal ($\dot{\xi}=0$), θ has increased an amount

$$\delta_{1}\alpha = -\frac{2\pi}{p} \frac{\mu \sin 2\alpha}{\sin^{2}\alpha + 4\cos^{2}\alpha}, \quad . \quad . \quad (11)$$

which corresponds to $\delta \alpha$ in the previous case. Thus the path of the particle is a downward spiral.

5. Consider now, briefly, the case where the friction is proportional to the normal reaction instead of to the velocity.

Eliminating the normal reaction from the equations of motion, we obtain as equations corresponding to (1) and (2)

$$\begin{split} \ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta &= -\frac{g}{a} \sin \theta - \frac{\mu \dot{\theta}}{v} \left(v^2 + \frac{g}{a} \cos \theta \right), \\ \frac{1}{\sin \theta} \frac{d}{dt} \left(\dot{\phi} \sin^2 \theta \right) &= -\frac{\mu \dot{\phi} \sin \theta}{v} \left(v^2 + \frac{g}{a} \cos \theta \right), \end{split}$$

where μ now denotes the (ordinary) coefficient of friction, and where $v^2 = \dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta$, the + sign being taken in front

of the square root for v. The expression $v^2 + \frac{g}{a}\cos\theta$ is

proportional to the normal reaction inwards and is ordinarily positive; if, however, the conditions allow of it becoming negative, the sign must be changed when this occurs. These equations hold so long as $v\neq 0$, but if v=0 at any instant when $\theta=\theta_0$, then the particle will remain permanently at rest unless $\tan\theta_0 > \mu$.

Using the same notation, (3) and (4) may now be written.

$$\frac{dh}{dt} = -\mu h \frac{2E + 3\frac{g}{a}\cos\theta}{\sqrt{2\left(E + \frac{g}{a}\cos\theta\right)}},$$

$$\frac{dE}{dt} = -\mu \left(2E + 3\frac{g}{a}\cos\theta\right)\sqrt{2\left(E + \frac{g}{a}\cos\theta\right)}.$$
(12)

Since the friction always opposes motion and does not now tend to zero with the velocity, it is evident that after a *finite* interval of time the particle will come to rest somewhere within the cone $\theta = \tan^{-1}\mu$, and not necessarily in the lowest position, as in the previous case.

If μ is small we can again integrate equations (12) approximately, the approximation holding for an interval of time not too large. The integrals can be briefly written

$$h = h_0 \left(1 - \mu \int_0^t \frac{\mathbf{R}}{v} dt \right); \quad \mathbf{E} = \mathbf{E}_0 - \mu \int_0^t \mathbf{R} v dt,$$

where Ra is the normal reaction per unit mass, and the integrals are calculated on the hypothesis of frictionless motion; they can be expressed as elliptic integrals.

By a precisely analogous method we now obtain in place of (9) and (10), and using the same notation,

$$\delta \alpha = \frac{-2\mu}{\sin \alpha \left(1 - \frac{a}{g} \Omega_{\alpha}^{2} \cos \alpha\right)} \int_{0}^{T} \frac{R}{v} (\cos \theta - \cos \alpha) dt,$$

$$\delta \beta = \frac{-2\mu}{\sin \beta \left(\frac{a}{g} \Omega_{\beta}^{2} \cos \beta - 1\right)} \int_{0}^{T} \frac{R}{v} (\cos \beta - \cos \theta) dt.$$

These differ from (9) and (10) only in the inclusion of the factor $\frac{R}{v}$ in the two integrals, but μ has not, of course, the same significance. For the "conical pendulum" case we find corresponding to (11) by a similar method

$$\delta_{1}\alpha = -\frac{2\pi}{p} \frac{2\mu \sqrt{\frac{g}{a}\cos\alpha}}{\sin^{2}\alpha + 4\cos^{2}\alpha}$$

Rotating Sphere.

1. Let ω denote the angular velocity of the sphere about the vertical diameter, and assume a resistance per unit mass equal to μ times the *relative* velocity. Then θ and ϕ denoting as before the coordinates of the particle in space (ω being measured in the same sense as ϕ), the equations of (absolute) motion are identical with (1) and (2) above, except that in the right-hand side of (2) we must replace ϕ by $\phi - \omega$. In place of (3) and (4) we then have

$$\frac{dh}{dt} = -\mu(h - \omega \sin^2 \theta),$$

$$\frac{dE}{dt} = -\mu \left(2E + \frac{2g}{a}\cos\theta - h\omega\right).$$
That E and h do not now necessarily decrease.

We see at once that E and h do not now necessarily decrease; whether, at any time, they do so or not depends on the value of ω . Evidently the particle tends asymptotically to a position of relative rest, when θ and $\phi(=\omega)$ will be constant. Now, it is easily seen from (1) that there are two possible positions of relative equilibrium given by $\theta=0$, or

 $\cos \theta = \frac{g}{a\omega^2}$, the second existing provided $\omega^2 > \frac{g}{a}$. But from the appropriate condition for "permanent" stability, namely

$$V - T_0 = minimum *$$

^{*} Lamb, 'Hydrodynamics,' p. 291.

we see that the position $\theta = 0$ is unstable if the second exists. Therefore the particle tends asymptotically to the bottom of the sphere or to a position of relative rest at

$$\theta = \cos^{-1} \frac{g}{a\omega^2},$$

according as ω^2 is less or greater than $\frac{g}{a}$ *.

2. Although it does not appear possible to obtain any first integral of the equations, we can still regard the motion in space as being of the spherical pendulum type, the constants defining this motion slowly changing, provided μ is small and ω is not too large. We can then integrate equations (13) approximately for a limited period of time; the solu-

tions involve the integrals $\int_0^t \cos\theta \, dt$ and $\int_0^t \sin^2\theta \, dt$, which can both be expressed explicitly in terms of t by means of elliptic functions. We thus find in place of (9) and (10)

$$\begin{split} \delta \alpha &= \frac{-2\mu}{\sin \alpha \left(1 - \frac{a}{g} \, \Omega_{a}^{2} \cos \alpha\right)} \\ & \left[2(\mathbf{I} - \mathbf{T} \cos \alpha) - \frac{a}{g} \omega \Omega_{a}(\mathbf{T} \sin^{2} \alpha - \mathbf{I}') \right], \\ \delta \beta &= \frac{-2\mu}{\sin \beta \left(\frac{a}{g} \, \Omega_{\beta}^{2} \cos \beta - 1\right)} \\ & \left[2(\mathbf{T} \cos \beta - \mathbf{I}) - \frac{a}{g} \, \omega \Omega_{\beta}(\mathbf{I}' - \mathbf{T} \sin^{2} \beta) \right], \end{split}$$

where the notation is the same as in (9) and (10), and where

$$I' = \int_0^T \sin^2 \theta \, dt \; ;$$

 Ω_{α} , Ω_{β} refer to absolute values. I and I' may both be expressed as elliptic integrals. In this case, then, $\delta \alpha$ and $\delta \beta$ may have either sign; if ω be sufficiently large—we must

* The instability in the lowest position if ω exceeds $\sqrt{\frac{g}{a}}$ exhibits a fundamental difference that may arise in the question of stability owing to the presence of friction. See Lamb, "On Kinetic Stability," Proc. Roy. Soc. lxxx. (1908), where this is discussed and other examples are given.

not, of course, suppose it so large that these approximations no longer hold—both circles of latitude will now rise in the sphere.

3. We can show that if $\delta \alpha > 0$ then $\delta \beta < 0$, and consequently if $\delta \beta < 0$ then $\delta \alpha < 0$. For we can write

$$\begin{split} \delta \alpha &= \mathbf{K} \! \int_0^{\mathbf{T}} \! \left(\cos \theta \! - \! \cos \alpha \right) \! \left[\omega \Omega_a \! \left(\cos \alpha \! + \! \cos \theta \right) \! - \! \frac{2g}{a} \right] dt, \\ \delta \beta &= \mathbf{K}' \! \int_0^{\mathbf{T}} \! \left(\cos \beta \! - \! \cos \theta \right) \! \left[\omega \Omega_\beta \! \left(\cos \beta \! + \! \cos \theta \right) \! - \! \frac{2g}{a} \right] dt, \end{split}$$

where K, K' are positive constants. Hence if $\delta \alpha > 0$ we must have

$$\omega > \frac{2g}{a\Omega_a(\cos \alpha + \cos \beta)} = \Omega_{\beta},$$

for otherwise the integrand would always be negative. Similarly, if $\omega > \Omega_a$ then $\delta \beta$ is certainly positive. But $\Omega_{\beta} > \Omega_a$, and hence the result is proved. It is easy to see, however, that if $\delta \alpha < 0$ then $\delta \beta$ may have either sign.

Since
$$\Omega_{\alpha}^{2} < \frac{g}{a} \sec \alpha$$
, we see that if $\cos \alpha > \frac{g}{a\omega^{2}}$ then $\delta \beta > 0$;

and similarly, if $\cos \beta < \frac{g}{\alpha \omega^2}$ then $\delta \alpha < 0$. In other words,

if the upper circle of latitude of the frictionless motion lies above the ultimate relative equilibrium position, the lower circle will be rising; while if the lower circle lies below this position, the upper circle will be falling. In particular,

if $\omega^2 < \frac{g}{a}$, so that the bottom position is the only one of rela-

tive equilibrium, the upper circle will always fall, but not necessarily the lower one. We may observe that if Ω_{α} (and hence also Ω_{β}) is negative—*i. e.*, the particle is circulating in the opposite direction to the rotation of the sphere,—then $\delta \alpha$, $\delta \beta$ are necessarily negative.

As a particular case consider that in which $\Omega_{\alpha}=0$, that is, in the absence of friction the motion would be in a fixed plane. In this case $\delta \alpha$ is negative, but the formula for $\delta \beta$ fails, as we would expect, for then $\beta=0$ and $\Omega_{\beta}=0$. We may then proceed as follows. Since $h_0=0$, h is obviously small. Hence from (13) we may write

$$h = \mu \omega \int_0^t \sin^2 \theta \, dt \; ; \quad \mathbf{E} = \mathbf{E}_0 (1 - 2\mu t) - \frac{2\mu g}{a} \int_0^t \cos \theta \, dt.$$

If $\theta = \epsilon$ is the lowest position in the sphere reached, we have for the energy there

$$\frac{1}{2}\frac{h^2}{\sin^2\epsilon} - \frac{g}{a}\cos\epsilon = \mathbb{E}_0(1-2\mu T') - \frac{2\mu g}{a} \int_0^{T'} \cos\theta \, dt$$

where T' is $\frac{\mathrm{T}}{2}$ in the previous notation, i. e., one-quarter of the complete period of a simple pendulum of angle α . Substituting for h, since $\mathrm{E}_0 = -\frac{g}{\alpha}\cos\alpha$, this gives

$$\begin{split} \frac{\mu^2 \omega^2}{2 \sin^2 \epsilon} \Big[\int_0^{\mathrm{T}'} \sin^2 \theta \, dt \Big]^2 + \frac{2 \mu g}{a} \int_0^{\mathrm{T}'} \cos \theta \, dt + 2 \mu \mathrm{E}_0 \mathrm{T}' \\ = -\frac{g}{a} \cos \alpha + \frac{g}{a} \cos \epsilon. \end{split}$$

Since on the right-hand side there is a term $\frac{g}{a}(1-\cos\alpha)$ which is not small, we can equate this to the first term on the left-hand side, giving approximately

$$\epsilon = \frac{\mu \omega}{2\sqrt{\frac{g}{a}\sin\frac{\alpha}{2}}} \int_0^{\mathbf{T}} \sin^2\theta \, dt.$$

This gives the small deviation from the position $\theta=0$ produced by the friction. What we have previously called $\delta\beta$ is twice this.

For the "conical pendulum" case we find, corresponding to (11),

$$\delta_{1}\alpha = -\frac{2\pi}{p} \frac{\mu \sin 2\alpha}{\sin^{2}\alpha + 4\cos^{2}\alpha} \left(1 - \omega \sqrt{\frac{a\cos\alpha}{g}}\right),\,$$

showing that the particle ascends or descends according as ω^2 is greater or less than $\frac{g}{a\cos\alpha}$.

4. In the case where the ordinary laws of friction are assumed and the coefficient of friction is small, we can easily obtain expressions for $\delta \alpha$, $\delta \beta$, $\delta_1 \alpha$ analogous to the above. They are somewhat complicated, however, and it does not seem worth while to reproduce them. In the general case, just as in the case of the fixed sphere, we see that after a finite time the particle will come to relative rest within certain regions on the sphere.

There is in any case a region of relative equilibrium below a certain circle of latitude, while if ω is sufficiently great, there is, in addition, another such region between two circles of latitude, both lying above the first-mentioned circle. In this case, since the particle can rest in relative equilibrium when displaced a small distance from the lowest position, we cannot say that this position is unstable, even if the angular velocity of the sphere is very large.

I wish to express my thanks to Professor Bartky, of the University of Chicago, who kindly gave me the benefit of his criticism during the writing of this paper.

University of Toronto.

LXVIII. The Eleven-year and Nineteen-year Periods and other related Periods of Earthquake Frequency. By CHARLES DAVISON, Sc.D., F.G.S.*

MOST of the results of this paper are based on the late Prof. Milne's great catalogue of more than four thousand destructive earthquakes between the years A.D. 7 and 1899†. In estimating the intensity of the shocks, Milne used the following scale:—

- Walls cracked, chimneys broken, or old buildings shattered.
- 2. Buildings unroofed or shattered and some thrown down,
- 3. Towns destroyed and districts desolated.

By taking 3-yearly means of the annual numbers of earthquakes of each intensity, minor inequalities in the frequencycurve are smoothed away, and the years in which earthquakes are clustered stand out clearly. These years are given in a short paper recently published ‡, and it is there shown that some of the intervals between the cluster-years are about 11 years or multiples of 11 years, and that many of the cluster-years agree closely with years of low sun-spot frequency.

^{*} Communicated by the Author. † Brit. Ass. Rep. 1911, pp. 649-740. † 'Nature,' cxx. pp. 587-588 (1927).

The 11-year Period.

In the present paper, I have used a simple form of harmonic analysis, that gives results of sufficient accuracy considering the nature of our earthquake-catalogues*. For the 11-year period, the numbers of earthquakes in the years 1701, 1712, ..., 1888; 1702, 1713, ..., 1889; and so on, were added together, and 5-yearly means were taken of the 11 sums. By this process of taking means, the amplitude of the period is reduced and that obtained must be multiplied by the factor 1.462. In all cases, unless otherwise mentioned, the earthquakes are those of the Northern Hemisphere (2421 in number from 1701 to 1898), the records for the Southern Hemisphere (303 earthquakes) being too scanty to give results.

TABLE I.

Intensity.	Max. Epoch.	Amplitude.
3	1709	·16
2	1708 -	· 18
1	1708-09	•09
3, 2, 1	1708-09	•12

As an 11-year earthquake-period is one of some interest, it seemed desirable to test its reality in other ways. In Table II. are given the results for different centuries; in Table III. those for different seasons; and in Table IV. those for different regions, the three intensities being grouped together in Tables III. and IV.

TABLE II.

Intensity.	Interval.	Max. Epoch.	Amplitude.
	{ 1701–99 1801–99	1709 · · ·	•39
3	1801-99	1807 (1708)	•09
	(1701–99	1709	· 2 2
2	{ 1701–99 1801–99	1807 (1708)	•16
	f 1701–99	1708	•04
1	{ 1701–99 1801– 9 9	1808 (1709)	·12
	ſ 1701-99	1709	·15
3, 2, 1	{ 1701-99 1801-99	1807 (1708)	•12

^{*} An account of the method is given in Phil. Trans. 1893 A, pp. 1108-1111.

TABLE III.

Season.	Max. Epoch.	Amplitude.
Winter (DecFeb.)	1710	·19
Spring (MarMay)	1708	•18
Summer (June-Aug.)	1707	•20
Autumn (SepNov.)	1709	•09

TABLE IV.

Region.	Max. Epoch.	Amplitude.
Europe	1709	- 15
Asia	. 1708	-18
Italy	1709	٠20
China	1709-10	•22
Island groups of W. Pacific	1708-09	·20

The following results are of less value, owing to the comparatively small numbers of earthquakes recorded: North America, epoch 1702, amplitude '25; Central America, epoch 1708, amplitude '38; West Indies, epoch 1709, amplitude '18.

The results given in Table V. are of interest, as most of

the earthquakes did not reach destructive intensity.

TABLE V.

Region.	Interval.	Max. Epoch.	Amplitude.
Italy (Baratta)	1701-1898	1707-08	•12
Great Britain (Davison)	23	1708-09	•32
Switzerland (Swiss Earthq. Ser.)	1883-1926	1885 (1709)	•37
Norway (Kolderup)	1893-1925	1895 (1708)	•28
Zante (Barbiani)	1826-58	1831 (1710)	.73
Philippines (Masò)	1801-1921	1807 (1708)	•34
New Zealand (Hogben)	1868-89	1875 (1710)	·10

The 22-year Period.

With the aid of Milne's catalogue, I next tested the existence of other periods of various lengths. In addition to two or three possible periods, there are four periods of some importance, of durations 22, 33, 19, and 38 years. These were traced by taking 11-year means of the 22 sums, 17-year means of the 33 sums, 9-year means of the 19 sums,

and 19-year means of the 38 sums, the angmenting factors being 1.576, 1.624, 1.501, and 1.574, respectively. The reality of these periods was tested in the same way as that of the 11-year period, except that for two periods—those of 22 and 38 years—earthquakes of intensity 1 were omitted.

TABLE VI.

Intensity.	Interval.	Max. Epoch.	Amplitude.
3	1701–1898	1716	•24
2	29	1719	·14
3, 2	,,	1716	·17
3	f 1701–1810	1717	•27
•••••••	1789-1898	1804 (1716)	•22
2	§ 1701–1810	1715	•25
~ ************	1789–1898	1807 (1719)	·16
0.0	f 1701–1810	1716	·17
3, 2	1789-1898	1804 (1716)	•16

TABLE VII.

Season.	Max. Epoch.	Amplitude.
Winter (DecFeb.)	1714	'24
Spring (MarMay)	1717	·13
Summer (June-Aug.)	1718	•30
Autumn (SepNov.)	1716	· 2 0

TABLE VIII.

Region.	Max. Epoch.	Amplitude.
Europe	1719	•20
Asia	1716	•24
Italy	1719-21	· · · · ·3 6
China	1716	33
Island groups of W. Pacific	171819	· 2 5

The following results are again of less value owing to the comparatively small numbers of earthquakes: North America, epoch 1715, amplitude '41; West Indies, epoch 1717-19, amplitude '27.

For Italy (Baratta), the epoch was 1720 and the ampli-

tude .24.

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The 33-year Period.

TABLE IX.

Intensity.	Interval.	Max. Epoch.	Amplitude.
3	1701-1898	1724-29	·13
2	9)	1724	•29
1	,,	1724	•28
3, 2, 1	,,	1724	•26
3, 2, 1	∫ 1701–99	1725	•36
U, 22 1	1801-99	1823 (1724)	-24

TABLE X.

Season.	Max. Epoch.	Amplitude.
Winter (DecFeb.)	1724	•37
Spring (MarMay)	1725	•21
Summer (June-Aug.)	1723	•24
Autumn (SepNov.)	1724	29

TABLE XI.

Region.	Max. Epoch.	Amplitude.
Europe	1724	·2 3
Asia	1724	•31
Italy	1725	·2 6
Island groups of W. Pacific	1724	•41

For the destructive earthquakes of North America, Central America, and the West Indies, the epoch was 1721 and the amplitude '28. For the earthquakes of Italy (Baratta) the epoch was 1724-25 and the amplitude '24.

The 19-year Period.

TABLE XII.

Intensity.	Interval.	Max. Epoch.	Amplitude.
3	1701-1890	1717	•29
2	31	1715	•20
1	,	· · · · · 1714 · · · · · · · ·	•24
3, 2, 1	,,	1714	•21
3	§ 1701–95	1715	•24
	l 1801–95	1812 (1717)	· 3 3
2	∫ 1701–95	1715	•26
AC D000000000000000000000000000000000000	l 1801-95	1813 (1718)	·12
1	{ 1701–95	1714	• 29
A	l 1801–95	1808 (1713)	15
3, 2, 1	§ 1701–95	1714–1 5	•23
	l 1801-95	1809 (1714)	·12

TABLE XIII.

Season.	Max. Epoch.	Amplitude.
Winter (DecFeb.)	1717	•24
Spring (MarMay)	1714	.23
Summer (June-Aug.)	1714	·15
Autumn (SepNov.)	1714	· • 2 9

TABLE XIV.

Region.	Max. Epoch.	Amplitude.
Europe	1714	23
Asia	1716	• 2 3 ·
Italy	1714	-11
China	1716	•39
1sland groups of W. Pacific	1717	• 15

For the destructive earthquakes of North America, the epoch was 1716 and the amplitude 29. For the earthquakes of Italy (Baratta, 1701-1890), the epoch was 1714 and the amplitude 11.

The 38-year Period.

In the following table the results are given for earthquakes of intensities 3 and 2 together, the duration of the record in each case being from 1701 to 1890:—

TABLE XV.

Region.	Max. Epoch.	Amplitude.
North Hemisphere	. 1724–25	•19
" Winter (DecFeb.)		doment
", ", ", Spring (MarMay)	. 1723	•31
", ", Summer (June-Aug.)	. 1725	•30
,, ,, Autumn (SepNov.)	. 1722	.27
Europe	. 1723	·17
Asia	. 1724	.27
Italy	. 1721-22	·31
China	. 1724	•39
Island groups of W. Pacific	. 1725	-24

Periodicity and Latitude.

Table XVI., in which earthquakes of intensities 3 and 2 from 1701 to 1898 are combined, shows that, in the Northern Phil. Mag. S. 7. Vol. 7. No. 43. March 1929.

Hemisphere, the periodicities do not depend on the latitude *.

TABLE XVI.

Period.	North T	ropics.	North Tempe	erate Zone.
years.	Max. Epoch.	Amplitude.	Max. Epoch.	Amplitude.
11	1708	•3ŏ	1709	•15
22	1716-17	•32	1716	•11
33	1724-25	·18	1722–24	•24
19	1715	20	1715	•21
3 8	1726-28	•33	1723-24	•17

Conclusions.

(i.) In the earthquakes of the Northern Hemisphere, there are periods of 11, 22, 33, 19, and 38 years, with maximum epochs in 1709, 1716, 1724, 1715-16, and 1724-25, respectively †.

(ii.) All over the Northern Hemisphere the maximum

epoch of each period is approximately the same.

(iii.) The periods of 11, 33, and 19 years affect similarly the destructive earthquakes of each intensity. The periods of 22 and 38 years are apparently confined to destructive earthquakes of intensities 3 and 2 only.

LXIX. High-frequency Discharges in Helium and Neon. By R. L. HAYMAN, B.A., D. Phil., New College, Oxford 1.

- 1. THE properties of electrical discharges produced by undamped high-frequency oscillations have formed the subject of several investigations. Hulburt § in America, and Gutton | sand his collaborators in France, measured the least potentials required to start these discharges in different gases, using a large range of oscillation frequency.
- * In the South Tropics and the South Temperate Zone, the epochs of the 11-year period are 1705 and 1703, those of the 19-year period are 1715-16 and 1706.
- † In the Southern Hemisphere, taking the earthquakes of intensities 3 and 2 only (118 in number), the corresponding epochs are 1703, 1718, 1723, 1710, and 1715.

† Communicated by Prof. J. S. Townsend, F.R.S. § E. O. Hulburt, Phys. Rev. vol. xx. p. 127 (1922). © C. Gutton, Journ. Phys. t. 4, pp. 420-429 (1923); also C. R. t. 186, p. 303 (1928).

Kirchner * and Gill & Donaldson † discovered that the potentials required to maintain these discharges were much smaller than those required to maintain direct-current discharges under similar conditions, especially when the pressure of the gas was low and the frequency of oscillation very high. Townsend and Donaldson ‡ studied discharges in bulbs and cylinders, using movable external electrodes in the form of sleeves. They showed that the electric force (volts per centimetre) required to maintain the glowing column of gas, in the cylindrical tubes, was independent of the distance between the electrodes. Townsend § subsequently found that the value of this force was independent both of the amplitude of the current and of the frequency of oscillation, and he has developed a theory in agreement with these properties of the discharges

Richards | and Hiedemann ¶ have observed striated forms of discharge in hydrogen. These have been carefully examined by Hiedemann, who has also made a quantitative study of the properties of high-frequency discharges in hydrogen and a qualitative study of the discharge phenomena

in air, oxygen, nitrogen, argon, etc.

In the present paper I propose to describe some measurements of the potentials required to start and maintain high-frequency discharges in helium and neon in cylindrical The principal object of the experiments was to determine the changes in the potentials due to changes in the pressure of the gas, and in the diameter of the tube, for oscillations of different wave-lengths.

2. The arrangement of the apparatus for purifying the helium and neon was the same as that described in the accounts of other researches which were made in this laboratory **. Traces of hydrogen were removed by passing the gas over copper oxide at a dull red heat, and other impurities were absorbed by keeping the gas in a tube containing charcoal, which was cooled by liquid air. Suitably placed liquid-air traps prevented contamination by mercury vapour from the McLeod gauge or evacuation

^{*} F. Kirchner, Ann. der Phys 1xxvii. (3) p. 287 (1925).

⁺ E. W. B. Gill and R. H. Donaldson, Phil. Mag. ii. p. 129 (1926).

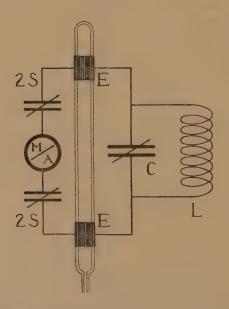
[†] J. S. Townsend and R. H. Donaldson, Phil. Mag. v. p. 178 (1928), § J. S. Townsend, C. R. t. 186, p. 55 (Jan. 1928). R. C. Richards, Phil. Mag. ii. p. 508 (1926).

[¶] Hiedemann, Ann. der Phys. lxxxv. p. 43 (1928). ** J. S. Townsend and C. M. Focken, Phil. Mag. ii. p. 474 (1926); L. G. Huxley, Phil. Mag. v. p. 721 (1928)

pumps. The purity of the gas could be tested by a spectroscopic examination of the high-frequency discharge, using a direct-vision spectroscope.

3. The discharge-tubes were cylindrical, and were fitted with movable external electrodes. The tubes were thoroughly baked out before use, and were frequently re-heated in the course of the experiments. This was done while they contained some helium or neon, which was pumped out while the tubes were still hot. Any impurities given off

Fig. 1.



by the walls were thus removed. One tube was made of quartz and the others of pyrex. It was possible to bake out the quartz tube at a much higher temperature than the pyrex tubes. The results of the experiments on the measurements of potentials of discharges in quartz and pyrex tubes were exactly the same when all the tubes had been heated to as high a temperature as possible for many hours.

4. The electrical arrangements for measuring the potentials were similar to those developed by Townsend and Donaldson. The source of high-frequency oscillations was a 60-watt valve generator. This was loosely coupled to the coil L (fig. 1)

of an oscillatory circuit (L: C and S) tuned approximately to resonance. The electrodes (E) in the form of lead sleeves, 4 cm. long, fitting closely to the surface of the tubes, were connected to the ends of the coil (L). The amplitude of the potential between the electrodes could be measured in terms of the current passing through the two equal condensers (2 S) shunted across the main tuning condenser (C).

The measuring instrument was a low-resistance thermal ammeter reading from 10 to 100 milliamps. It was connected between the two condensers (2 S), and it was therefore at a potential node. This ensured that its readings

were unaffected by its capacity to earth.

5. The radial distribution of luminosity in the discharge depends on the pressure of the gas and the current flowing between the electrodes. The ratio of the intensity near the axis to that near the walls of the tube is greatest at high pressure and in weak discharges, and it appears to increase

with the wave-length of oscillation.

The colour of the discharge varies with the pressure of the gas and the intensity of the electric field. When the pressure of the gas is high and the field-strength low, the lines towards the red end of the spectrum seem to be the strongest. The result of decreasing the pressure or increasing the electric field is to shift the region of maximum intensity towards the blue end of the spectrum. Thus the colour of the discharge in neon at 80 mm. pressure is orange under the electrodes where the electric force is strong, and bright red throughout the rest of the tube. At $\frac{1}{3}$ mm. pressure the ratio of the electric force to the pressure is greater, and the discharge is yellow throughout.

6. The potentials required to start and to maintain these discharges depend on several factors, of which the following are the most important:—The nature, purity, and pressure of the gas, the distance between the electrodes, the diameter of the discharge-tube, and the wave-length of the oscillations. Measurements of the smallest potentials required to start discharges could be repeated to an accuracy of 3 or per cent, under a given set of conditions. A higher degree of accuracy was obtainable in measuring the smallest potentials required to maintain a small current with a uniform glow throughout the whole length of the tube, between the electrodes.

In this paper these potentials will be referred to as the starting and the maintenance potentials.

Such variations as were observed were always greater when the distance between the electrodes was large than when it was small.

7. Quantities of impurity too small to be easily detected spectroscopically produced a marked effect on the starting potential. Thus a pyrex tube which had only received a few hours' heating was filled with pure neon, and the starting potentials were measured. A small area of the tube was heated with a blow-pipe until the glass began to soften. In the course of this heating the colour of the discharge changed from red to blue, owing to the evolution of watervapour, hydrogen, and other impurities. The tube was then allowed to cool, and the starting potentials were again measured with the same gas, which contained a small amount of impurity. The potentials were found to be smaller than those obtained when the gas was pure, even though the discharge had practically regained its original colour. The spectrum was then examined, and an extremely faint background due to impurities was observed.

The results of these experiments are given in Table I., where the amplitudes of the potentials are given in volts.

TABLE I.

Neon at 2.55 mm. pressure (measured before and after the tube had been heated). Tube: Pyrex 3.9 cm. diameter. Wave-length 80 metres.

Distance he electrode		Starting Potentials before heating.	Starting Potentials after heating.
3 cm.		232	198
6 ,,	*******	357	344
9 ,,	******	490	463
12 ,,	*******	566	518

The experiment was repeated, using helium in the place of neon, and the results obtained were exactly similar. In both gases the lowering of the maintenance potentials, resulting from the introduction of the impurity, was much less than the lowering of the starting potentials, and was too small to be determined with any degree of accuracy. This is attributable to the removal of impurities from the gas, by the passage of the discharge, in the manner described by Townsend and MacCallum *.

^{*} J. S. Townsend and S. P. MacCallum, Phil. Mag. v. p. 695 (1928).

8. The starting potentials of discharges in pure helium and neon were measured over a large range of pressures. It was found that there corresponds to each distance between the electrodes a certain pressure at which the starting potential is a minimum. The value of the potential at this minimum increases with the distance between the electrodes. The results of measurements of the starting potentials of discharges in a quartz tube 2.9 cm, in diameter are given in Table II., in which the starting potentials are given in terms of the pressure p and the distance x between the inner edges of the electrodes. The wave-length used was 80 metres.

TABLE II.

Starting Potentials in terms of p and x in Neon and Helium.

	Neon.		Helium.	
p, in mm.	w=3 cm. Volts.	x=6 cm. Volts.	w= 3 cm. Volts.	. x=6 cm. Volts.
0.25	242	260		
0.5	222	285	320	320
1.0	210	322	225	350
2.0	230	380	245	400
4.0	2 90	455	285	480
8.0	385 -	570	345	590
16.0	485	740	440	755
32. 0	490	880		_

- 9. Experiments were also made with tubes of different sizes. It was found that the greater the diameter of the tube the smaller the starting potential: for example, measurements were made using tubes of 1.5, 2.9, and 3.9 cm. diameter (when the electrodes were 3 cm. apart). The starting potentials in the 1.5-cm. tube exceeded those in the 3.9-cm. tube by an amount which was small at high pressures (not more than 20 per cent. at 30 mm.), but which increased as the pressure was lowered, and reached as much as 100 per cent. at about 1 mm.
- 10. The starting potentials were also determined for oscillations of different wave-lengths from 40 and 640 metres. Between these limits the potentials increase with the wavelength. The results of the experiments with the tube 2.9 cm. diameter are given in Table III. for wave-lengths (λ) of 40, 80, 160, and 320 metres, and pressures (ρ) from 1 mm. to

6 mm. At higher pressures the starting potential was approximately the same for all wave-lengths. The distance between the inner edges of the electrodes was 6 cm. in these experiments.

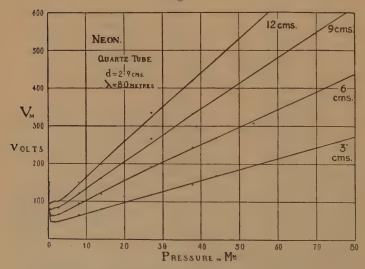
TABLE III.

Showing Variation in Starting Potential due to change of Wave-length.

N	Δ	^	n	
77	C	U	11	0

			λ.	
p,	40 metres.	80 metres.	160 metres.	320 metres.
in mm.	Volts.	Volts.	Volts.	Volts.
0.11	179	340	1000	1150
0.24	228	259	42 8	510
0.54	258	283	338	384
1.2	320	335	3 62	376
2 ·5	400	403	410	421
6.1	510	510	. 526	52 0

Fig. 2.

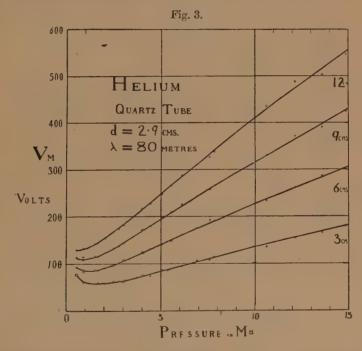


11. The potential required to maintain a discharge varies with the current. There is a certain current i_m for which the potential has a minimum value V_m . The current at this point is slightly greater than the least current which gives a uniform glow in the tube. The current i_m appears to be independent of the distance between the electrodes. The

-corresponding potentials V_m between the electrodes can be

readily measured.

The results of the experiments made with a tube 2.9 cm. in diameter containing neon at various pressures are given by the curves (fig. 2), the wave-length of the oscillations being 80 metres. In this diagram the ordinates represent the amplitudes of the potentials V_m in volts, and the abscissæ the pressure of the gas in millimetres. The four curves correspond to the distances 3, 6, 9, and 12 cm. between the electrodes.



The results of similar experiments with helium are given by the curves (fig. 3).

With each gas there is a certain value of the pressure for

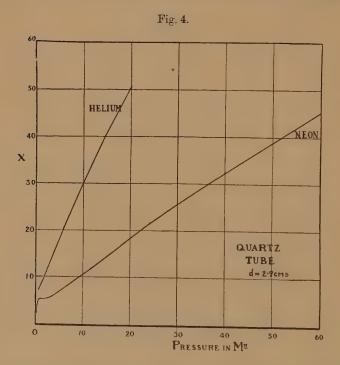
which V_m has a minimum value.

For higher pressures the potential increases with the pressure by amounts which are approximately proportional to the increases of the pressure, as is seen by the curves, which are approximately straight lines for the larger values of p.

An important characteristic of these discharges is that equal increases in the distance (x) between the electrodes

result in equal increases in the potential. Thus the electric force $X = \frac{V_m - V_{m'}}{x - x'}$ in the central part of the tube is

independent of the distance between the electrodes, as was observed by Townsend and Donaldson. The magnitude of this force depends on the pressure of the gas, and this is shown by the curves in fig. 4, where the ordinates give the amplitudes of the force X in volts per centimetre, and the



abscissæ the pressure in millimetres. The values of X were found to be independent of the wave-length of the oscillations within the range from 40 to 640 metres.

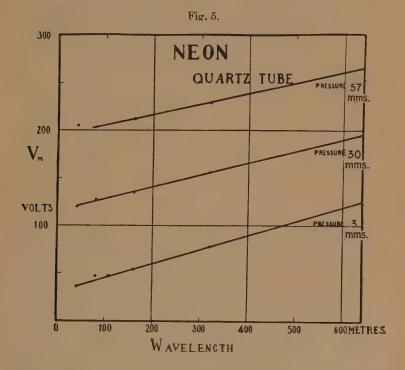
12. The maintenance potentials vary considerably with the wave-length, which shows that the potential between the electrodes consists of two parts.

The first is the fall of the potential in the central column of the gas, in which the electric force is uniform. This potential is independent of the wave-length, and is equal to the product of the electric force X and the effective length of the discharge *.

The second is the drop of potential at the electrodes, which is practically independent of the length of the discharge,

and is roughly proportional to the wave-length.

In fig. 5 is shown the variation of the maintenance potentials with the wave-length in neon. The three curves refer to three different pressures: 3, 30, and 57 mm. The ordinates



represent the amplitudes of the potentials in volts, and the abscissæ the wave-lengths. The diameter of the tube was 2.9 cm., and the electrodes were 3 cm. apart.

- 13. Both the maintenance potentials (V_m) and the electric forces X are greater in narrow tubes than in wide tubes. The values of these quantities for three tubes containing
- * The effective length of the discharge exceeds the distance between the inner edges of the electrodes by an amount which depends on the diameter of the tube. In the 2.9-cm. tube this amount is about 1.4 cm. This can easily be verified from figs. 2 and 3.

helium at various pressures p are given in Table IV., where V_m is the amplitude of the force in volts per cm., p the pressure in millimetres, and x the distance between the electrodes in centimetres.

Table IV. Helium. $\lambda = 80$ metres.

		Tu 1.5 cm.	be diameter.		ibe liameter.		ube liameter.
p, in mm.	w, in cm.	$\overline{\mathbb{V}_{mullet}}$	X.	$\overline{\mathbb{V}_m}$.	X.	$\overline{\mathbf{V}_{m}}$.	X.
1.0	3	148.0	10.0	60.5	0.0	46.0	7.0
,, *****	9	210.0	10.3	113.3	8.8	87.8	70
2.5	3	95.0	17.5	63.0	11.8	58.0	10.7
,,	9	200.0	17.5	134.0	11.5	122.0	10 1
10.0	3	138.0	94.77	134.0	90.5	130.0	29.8
,,	9	346.0	34.7	318.0	30.7	309.0	200
15.0	3	183.0	45·3	185.0	42.5	187.0	41:3
,,	9	455.0	10'0	440.0	74'U	435.0	31.9

14. Some experiments were made with ionizing radiations (such as ultra-violet light and Röntgen rays from external sources) falling on the discharge-tube, but there was no appreciable effect observed.

Stable striated forms of discharge have been observed in both helium and neon. Their occurrence seems to be favoured by the use of large discharge currents, narrow tubes, long

wave-lengths, and certain conditions of pressure.

When using a wave-length of 80 metres and a tube 2.9 cm. in diameter, striations were only observed in helium at pressures exceeding about 12 mm., and in neon at pressures less than 2 mm.

The striated form of discharge in helium was similar to that observed in hydrogen, and consisted of a series of pairs of luminous disk with their planes perpendicular to the direction of the electric force. The striated discharge in neon consisted of a series of luminous ovals equally spaced along the axis of the tube.

My best thanks are due to Professor Townsend for his advice and the interest he has taken in this work.

LXX. The Raman Effect in some Organic Liquids. By S. Venkateswaran *.

[Plate XV.]

I. Introduction.

THE recent discovery by Prof. Raman + of a new type of secondary radiation offers an entirely new and very accurate and convenient method for measuring the infra-red frequencies of transparent media. It has been found by him that when monochromatic light is diffused by a liquid, the scattered light ceases to be monochromatic and several new lines and sometimes bands appear in the scattered spectrum. The difference in frequency between the incident line and a modified line generated by it gives a natural frequency of the molecule. In a single photograph the modified lines corresponding to all the infra-red frequencies can thus be obtained and measured accurately, the degree of accuracy being limited only by the width of the line. In this paper the characteristic infra-red frequencies of some fatty acids have been determined by this method and their values given.

II. Experimental Methods.

The experimental arrangements were similar to those described by Raman and Krishnan‡ and by the author § in their recent papers. Kahlbaum's acetic acid and Merck's extra pure propionic and butyric acids were further purified by repeated slow distillation in vacuo in large double bulbs. The bulb containing the purified liquid was immersed in a glass tank containing water, and light from a 3000 c.p. mercury arc lamp was condensed into the liquid with an 8-inch condenser. The spectrum of the transversely-scattered light was taken with a Hilger E 2 quartz spectrograph.

The plates were measured with a Hilger travelling micrometer, and the wave-lengths of the modified lines were calculated, taking the mercury arc lines as standards. The simplified Hartmann interpolation formula was used.

* Communicated by the Author.

Secondary Radiation," Ind. Journ. Phys. vol. ii. pt. iv. p. 399 (1978).

§ S. Venkateswaran, "Study of Raman Effect in Glycerine and Glycerine Water Mixtures," Ind. Journ. Phys. vol. iii. pt. i. p. 105 (1928).

[†] C. V. Raman, "A New Radiation," Ind. Journ. Phys. vol. ii. p. 387 (1928). See also C. V. Raman and K. S. Krishnan, 'Nature,' vol cxxi. p. 501 (31st March, 1928).
† C. V. Raman and K. S. Krishnan, "A New Class of Spectra due to

III. Results.

The spectrograms of the light-scattering in acetic, propionic, and butyric acids are reproduced in figs. 2, 3, and 4 (Pl. XV.). (Fig. 1 represents the corresponding incident spectrum.) All the modified lines have been analysed, and the difference in wave-numbers between each of the exciting and its corresponding modified lines have been calculated. The characteristic infra-red frequencies thus obtained and their wave-lengths are given in Table I.

The modified lines corresponding to 7.0μ and 3.4μ broaden into a band as we go up the series. In fact, the band shows a fine structure which is, however, not well resolved in the instrument. The intensity of the modified line due to 16.0μ rapidly diminshes from acetic to butyric

acid.

The infra-red absorption spectrum of acetic acid has been studied by Coblentz, and that of butyric acid by Weniger *. A comparison of their values with those obtained by the

author is given in Table II.

The agreement between the values calculated from light-scattering and those obtained directly from infra-red measurements must be considered good. Of the various characteristic frequencies of the molecules of the fatty acids, the one corresponding to $3.4~\mu$ has been identified as corresponding to the C-H bond, and that corresponding to $7.3~\mu$

as arising from the O-H group.

An examination of the spectrograms reproduced in the paper shows that all the modified lines are rather broad and diffuse in the fatty acids, unlike those observed in benzene. toluene, carbon tetrachloride, and other liquids. In addition to the modified lines, there is a continuous spectrum. That the presence of the continuous spectrum is characteristic of the liquids and not due to any impurities, is shown by the fact that it is observable in these liquids, even after the most careful purification. An inter-comparison of the spectrograms shows that the intensity of the continuous spectrum is of the same order in acetic and propionic acids, but is appreciably larger in butyric acid. In his study of the Raman effect in glycerine the author has shown that the presence of the continuous spectrum, which is so prominent in this liquid. is connected with its high viscosity. On the basis of this hypothesis, the larger intensity of the continuous spectrum in the scattered light in butyric acid is easily understood in view of its higher viscosity.

^{*} See W. Weniger, Physical Review, xxxi. p. 388 (1910).

				T.A	BLE I.							
Acetic acid.	Shift in wave-number.	446	623	766	808	:	1128	1280	1364	1433	1670	2944
	Wave-length in μ	(<u>z</u>) 22·41	(9)	13.05	(c) 10·11	. :	(o) 8.8g	(3)	(3)	(3) 6-98	(2)	(4)
Propionic scid.	Shift in wave-number.	485	809	770	860	1040	1134	1275	1370	1443	1664	9955
	Wave-length in μ	(0)	(\$) 16•44	$(\frac{1}{2})$ 1 2.98	(4) 11.62	(0) 9.61	(3) 8:81	(1)	$(\frac{1}{2})$	(4) 6:93	(T) 6:01	(6)
Butyric acid.	Shift in wave-number.	:	:	278	869	1044	1125	. :	1364	1439	1659	9941
	Wave-length in μ	÷	÷	12.85	(2) 11·50	$(\frac{1}{2})$ 9.58	(1)	: :	(1)	(\frac{1}{2})	(\$)	(2)

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	7.33	7.33
	7.81	::
	8.86 8.9	88.88
	: :	9.58 8.0
ABLE II	11.01	11.50 10.6
L	13.05	12.85 12:9
	16.05	::
	$\begin{cases} \text{Wave-length in } \mu \text{ (author). } 22.41 \\ \text{,, (Coblentz)} \end{cases}$	$\left\{\begin{array}{ll} \text{Wave-length in } \mu \text{ (author).} & \dots \\ \text{``} & (\text{Weniger).} \dots \end{array}\right.$
	Acetic acid.	Butyric acid.

Summary.

- 1. The infra-red frequencies of acetic, propionic, and butyric acids have been measured from the Raman lines. The values for acetic and butyric acids agree closely with those obtained by Coblentz and Weniger. Some characteristic frequencies have been measured in the extreme infra-red region not accessible to their infra-red spectrometers.
- 2. The spectrum of light-scattering in the three acids shows the presence of the modified lines and a continuous spectrum. The intensity of the latter is nearly the same in acetic and propionic acids, but is appreciably larger in butyric acid. This larger intensity of the continuous spectrum is probably due to the higher viscosity of butyric acid.

The author desires to express his thanks to Prof. C. V. Raman, F.R S., for his kind guidance, and to Mr. K. S. Krishnan for his kind interest in the work.

210 Bow Bazar Street, Calcutta. September 6th, 1928.

- LXXI. High Frequency Discharges in Gases. By J. S. Townsend, M.A., F.R.S., Wykeham Professor of Physics, Oxford, and W. Nethercot, B.A., Merton College, Oxford*.
- 1. IN order to investigate certain properties of high frequency discharges in gases, it is convenient to use long cylindrical tubes with external electrodes in the form of sleeves which may be placed at different distances apart. By this means the potentials required to start electrodeless discharges have been measured, and also the minimum potential required to maintain the minimum current which gives a uniform glow in the tube †.

The investigations have been extended in order to determine the potentials required to maintain currents of different amplitudes, which give a uniform glow in the tube. With these currents the potential increases with the current,

^{*} Communicated by the Authors.

[†] J. S. Townsend and R. H. Donaldson, Phil. Mag. v. p. 178 (Jan. 1928).

but for a given current the rate of increase of the potential with the distance between the electrodes is a constant, which is the same over a large range of currents and over a large range of frequencies of oscillation.

This result, which was first obtained with tubes containing helium and neon, formed the basis of a theory * of the

uniform glow in an electrodeless discharge.

The theory indicates that the mean value of the electric force in the glow of a high frequency discharge should be the same as the force in the uniform positive column of a continuous current discharge.

- 2. The condition that a continuous current in a gas shall be maintained by a given field of force may be expressed in the form of a relation between the coefficients of ionization α and β which is independent of the intensity of the current †. Thus the potential is independent of the intensity of the current when the field of force is not disturbed by the current. This condition is obtained with small currents, since the charge in the gas (due to an excess of positive ions or electrons) is small and the field of force is not disturbed to any great extent. The charge in the gas increases with the current, and generally with large currents the field of force is altered and the force may become distributed so as to facilitate the discharge, as with currents between parallel plate electrodes, or the force may be distributed so as to impede the current as in the corona discharge. In these cases the charges which disturb the field are accumulated in the spaces near the electrodes.
- 3. At points in the gas remote from the electrodes the charge in the gas may be independent of the current, even when comparatively large currents are flowing. In these cases the electric force is independent of the current, provided the density of the gas in the path of the discharge is not changed to any great extent by the increase of temperature due to an increase of current.

The uniform positive column of a continuous current discharge in a long tube is an example of a space where the charge in the gas and the electric force parallel to the axis of the tube are constant over a large range of currents. The force parallel to the axis has been determined by Wilson over wide ranges of the currents. In air at pressures less than 1 mm. he found that the force was nearly

^{*} J. S. Townsend, Comptes Rendus, clxxxvi. p. 55 (Jan. 1928).

^{† &#}x27;Electricity in Gases,' section 293.

constant, but at higher pressures the force diminishes as the current increases. The experiments were made with a tube 2·1 mm. in diameter, and with air at 2·81 mm. pressure the force was independent of the current with currents from 10^{-4} to 10^{-3} ampere, but the force changes from 57 to 54 volts per cm. when the current is increased from $4\cdot05\times10^{-3}$ ampere to $7\cdot8\times10^{-3}$ ampere *.

4. We have obtained similar results in our experiments with high frequency discharges in nitrogen. With small currents the force in the uniform glow in the high frequency currents is constant, but with large currents the force changes with the current. The change of force with the current is much smaller in wide tubes [3 or 4 cm. in diameter] than in narrow tubes. This indicates that the change of force is probably due to the increase of temperature obtained with the large currents. An increase of temperature in the path of the discharge reduces the density of the gas, and a smaller force is required to maintain the current, since the force diminishes with the pressure when the temperature is constant.

In the positive column the ratio of the electric force to the pressure of the gas is small, and it may be assumed that positive ions do not ionize molecules of the gas, and that the current is maintained by the process of ionization due to collisions of electrons with molecules of the gas. The electric force is determined by the condition that the supply of ions due to this process is balanced by the loss due to diffusions to the sides of the tube. The theory formed on this hypothesis shows that there must be a positive charge in the gas which repels positive ions towards the surface and retards the rate at which electrons reach the surface by diffusion, so that the positive ions reach the surface at the same rate as electrons.

Thus the number of positive ions n_2 per cubic centimetre exceeds the number of electrons n_1 , but the excess (n_2-n_1) is small compared with n_2 or n_1 , and the charge $e(n_2-n_1)$ is independent of n_1 or n_2 . It is also necessary to assume that the effect of recombination of electrons and positive ions may be neglected, as is generally the rule in discharges at low pressures \dagger .

Thus there are two forces in the positive column: the force parallel to the axis, which is the force that has been determined experimentally, and a force along the radius due to

^{*} H. A. Wilson, Proc. Camb. Phil. Soc. ix. pp. 249 & 391 (1902). † 'Electricity in Gases,' section 302.

the charge $e(n_2-n_1)$ per c.c. of the gas. This charge has the effect of increasing the conductivity of the gas, since it reduces the rate at which electrons are lost by diffusion to the surface of the tube. There is also a charge on the inner surface of the tube which tends to become distributed so that the force along the axis is the same at all points of the uniform glow. This distribution on the surface of the tube regulates itself in the same way as the charge on the surface of a wire, which makes the electric force to be the same at all points when a continuous current is flowing in the wire.

5. On these principles a satisfactory explanation may also be obtained of phenomena which occur in the high frequency discharges in tubes with external electrodes. In the theory of these discharges which has recently been proposed, the charge $e(n_2-n_1)$ per unit length of the tube (which is independent of the current) is given in terms of the coefficients of diffusion and the mobilities of the positive ions and the electrons, and the coefficient of ionization.

The charge in the gas does not vary with the changes in direction of the oscillatory force parallel to the axis, so that

the electric force along the radius is not oscillatory.

The results of the investigation are consistent with the hypotheses:—that the coefficient of ionization a with large currents of the order 10^{-3} to 10^{-2} ampere is the same as that found with small photo-electric currents; the rate at which the electrons and positive ions are thus generated is the same as the rate at which they reach the sides of the tube; the rate of recombination of positive ions with electrons in the gas is very small, and the loss of conductivity due to this process may be neglected.

6. This paper contains an account of the methods that have been used to determine the relation between the current and the electromotive force in high frequency discharges. Most of the experiments were made on high frequency discharges in nitrogen, but some experiments were also made with continuous current discharges in this gas, with a view to determining the electric force in the uniform positive column, with tubes of the same diameter and containing gas at the same pressure as in the experiments with high frequency oscillations. The results show that there is an agreement between the values obtained for the forces in the continuous and high frequency discharges as indicated by the theory.

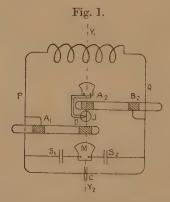
The nitrogen used in these experiments was prepared by heating pure sodium azide. The discharge-tubes were

2 R 2

evacuated to a very low pressure and heated in order to expel impurities from the surface of the glass. The tubes were then filled to a pressure of about 10 mm. with pure nitrogen, and the gas was pumped out so as to remove the impurities. This process was repeated several times, and the spectrum of an electrodeless discharge in the tubes was examined with each specimen of the gas. After the tubes had been washed out with pure gas three or four times, there were no lines due to impurities which could be seen with a direct-vision spectroscope.

When this stage was reached the tubes were allowed to cool, and were filled with nitrogen at the pressures required in the experiments on the determination of the high frequency

forces.



In most of these experiments the discharges were maintained by connecting the external electrodes to a secondary circuit which was loosely coupled with a continuous wave generator. The generators which were used were of the ordinary types where the wave-length is adjusted by changing the capacity of the oscillating circuit and the amplitude of the oscillation is adjusted by changing the heating current in the filaments of the valves. Small continuous changes of amplitude are thus obtained by means of a rheostat in the circuit connecting the filament with the low tension battery. Low-power transmitting valves were generally used with a low tension battery of 6 volts to heat the filaments, and a direct current generator of 800 volts to charge the anodes.

7. Experiments were made with several arrangements of the secondary circuit, and the symmetrical form which is shown in fig. 1 was found to be the most convenient and the most accurate for determining the current through the tube and the electromotive force between the electrodes. With this arrangement two discharge-tubes of the same dimensions and containing gas at the same pressure are required. The electrodes were in the form of sleeves A_1B_1 and A_2B_2 , which could be fixed at various distances apart.

The same current flows through the two tubes and through the heater of the thermojunction J, which is connected by short wires to the sleeve B_1 of one tube and the sleeve A_2 of the other. The current through the tubes is indicated by the microammeter I, which is connected to the thermojunction.

The circuit is symmetrical with respect to the plane through the line Y_1Y_2 with the axis of the inductance normal to the plane, and points in this plane remain at zero potential while the oscillatory current flows through the circuit. Thus the thermojunction and the microammeter are in positions of zero potential, and the positions of the sleeves B_1 and A_2 may be adjusted so that there is no current in the indicator when an oscillatory current flows in the circuit and the potentials at the sleeves A_1 and B_2 are not sufficiently high to start a luminous discharge in the tubes.

When the potentials are increased a discharge is obtained, and the current through the tubes is indicated by the micro-

ammeter.

8. The potentials at the sleeves A_1 and B_2 are measured by the current in the milliammeter M which connects the plates of the adjustable condensers S_1 and S_2 . These condensers were of small capacity, ranging from 15 to 120 cm., and were carefully calibrated. The capacities were adjusted to the same value S, which may be chosen so as to obtain a convenient deflexion in the milliammeter. The condensers were contained in metal cases which were connected together, both sets of plates in the condensers being insulated from the cases. A third adjustable condenser C was also used in order to bring the capacity of the circuit (C+S/2) near to the point of resonance with the generator.

A current of any required amplitude was obtained through the tubes by adjusting the capacity C or by the rheostat which controlled the intensity of the oscillations in the

generator.

The distance x between the sleeves A_1 and B_1 , and A_2 and B_2 , was adjusted by having flexible connexions to A_1 and B_2 . The sleeves were 4 cm. long, and were made to fit closely to the tubes.

The electromotive force between the sleeves is obtained

from the current M in the milliammeter by the formula E = M/p. S, where $p/2\pi$ is the frequency of the oscillations, E 12 being the amplitude of the electromotive force and M $\sqrt{2}$ the amplitude of the current.

9. Another arrangement of the secondary circuit which was used with single tubes in shown in fig. 2. In this case a large metal sheet P1P2 was used as a surface of zero potential, and the two measuring instruments were placed on the sheet.

The lower sleeve B of the discharge-tube was connected by short wires through the heater of a thermojunction J to the sheet P₁P₂, and the current through the tube was indi-

cated by the microammeter I.

One set of plates of the adjustable condenser S was connected through the low resistance milliammeter M to

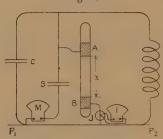
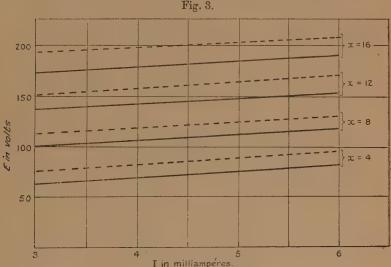


Fig. 2.

P₁P₂, so that the electromotive force between the sleeves A and B is given by the formula $E = M/S \cdot p$. As in the first arrangement, the capacity S was adjusted to give a suitable deflexion in the milliammeter, and the condenser C was used to bring the circuit into resonance with the generator. With this arrangement it was necessary to screen the microammeter and the sleeve B by conductors connected to P₁P₂, so as to have no induced current in the indicator before the current starts to flow in the tube.

In these experiments the heater of the thermojunction was a very fine wire of high resistance, as it was not required to carry currents exceeding one-fiftieth of an ampere. With oscillations of the wave-lengths that were used, the resistance does not differ by 1 per cent. from the direct current resistance, which was of the order of two ohms. Under these conditions, accurate measurements of the high frequency currents are obtained by calibrating the microammeter with a continuous current flowing through the heater of the thermojunction.

10. The circuit (fig. 1) was used with a pair of pyrex tubes 3.9 cm. in diameter containing nitrogen at .26 mm. pressure. The results of the experiments are shown by the four continuous curves (fig. 3), where the ordinates are proportional to the potentials, and the abscisse proportional to the currents, the wave-length of the oscillations being 40 m. In these diagrams the root mean square value of the potential E is given in volts and the current I in milliamperes, so that



Current-electronic-force curves for nitrogen at .26 mm. pressure,

Continuous curves refer to tube 3.9 cm. diameter. Dotted curves refer to tube 2.9 cm. diameter.

the amplitude of the potential is $\mathbb{E}\sqrt{2}$ and the amplitude of the current I $\sqrt{2}$.

The four curves correspond to the distances 4, 8, 12, and

16 cm. between the sleeves.

A similar set of experiments was made with the same pair of tubes, with the wave-length of the oscillations increased to 75 m. It was found that the rate of increase of E with the distance between the electrodes was the same as with 40 m., but the rate of increase of E with the current was greater in the proportion of 75:40.

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The values of E (in volts) obtained in these experiments may be expressed in terms of the distance x between the electrodes (in centimetres), the current I in milliamperes, and the wave-length λ in metres, by the formula:

$$\mathbf{E} = 8.9 \times x + .14\lambda \mathbf{I} + 11.$$

Table I. gives examples of the value of E obtained experimentally, and the values given by the formula for different values of x, λ , and I.

TABLE I.
Nitrogen '26 mm. pressure. Tube 3.9 cm. in diameter.

x.	I.	E expt.	E formula.	
4	3	64	63	
8	5	111	110	$\lambda = 40$
12	4	141	140	
16	6	191	187	
4	2	71	68	
8	5	138	135	λ = 75
12	4	161	160	
16	6	212	217	

With a tube 2.9 cm. in diameter containing nitrogen at the same pressure [.26 mm.] the electromotive force is given by the formula:

$$\mathbf{E} = 9.5 \times x + .17 \,\lambda \mathbf{I} + 11.$$

The value of E, for oscillations 40 metres wave-length, obtained with this tube are given by the four dotted curves (fig. 3) for the distances 4, 8, 12, and 16 cm between the electrodes.

11. In general the electromotive force E is given as the sum of two terms in the form:

$$\mathbf{E} = \mathbf{X}(x+a) + b\lambda \mathbf{I},$$

where a, b, and X are constants depending on the pressure

of the gas and the diameter of the tube.

The coefficient X (which represents the electric force parallel to the axis of the tube at points remote from the electrodes) depends on the pressure of the gas, but it is independent of the intensity of the current or the wavelength of the oscillations.

The term $b\lambda I$ represents an electromotive force in the spaces near the electrodes where the charges in the gas impede the current. It includes potential differences between the sleeves and the inner surfaces of the tubes, and potential differences in the gas near the electrodes where the

electric force differs from the value dE/dx.

There is no important term in the expression for E involving both I and x, since dE/dx is independent of I and dE/dI is independent of x. The experiments therefore indicate that the total electromotive force may be expressed as the sum of two potentials in series (represented by the two terms in the expression for E) which are approximately in the same phase.

12. The changes in the coefficients a, b, and X with the pressure are seen by comparing the values of E, obtained with a tube 3·1 cm. internal diameter containing nitrogen at ·53 mm. pressure, with the values of E for the same tube with nitrogen at the pressure ·26 mm. With the gas at ·53 mm. pressure the values of E are given by the formula:

$$E = 13.7 (x+4) + .21 \lambda I,$$

and with the pressure at .26 mm.:

$$E = 9.4 (x+1) + .165 \lambda I.$$

Thus the electric force X increases from 9.4 volts per cm. to 13.7 volts per cm. when the pressure is increased from 26 mm to 53 mm.

These values of X may be compared with the results of the earlier experiments on the force in the uniform positive column of a continuous discharge. In nitrogen at 1 mm. pressure in a tube 2.5 cm. in diameter the value of X found by Herz* was 26.9 volts per cm. Since the force diminishes when the pressure is reduced or the diameter of the tube increased, the value 13.7 volts per cm. obtained in a high frequency discharge with a tube 3.1 cm. in diameter containing nitrogen at .53 mm. pressure may be considered to be consistent with Herz's determinations.

[•] A. Herz, Wied. Ann. liv. p. 244 (1895).

This, however, needs further investigation, as the values of X for continuous current discharges found by different

observers are not in good agreement.

With large currents the force X and the coefficient b in the above formula for E tend to diminish as the current increases. This may be attributed to the reduction of the density due to the increase of temperature of the gas, since the electromotive force E diminishes when the density is diminished (by reducing the pressure). This explanation of the changes in X and b with large currents is in agreement with the fact that the changes are much more marked in narrow tubes than in wide tubes; also the changes are augmented when the pressure of the gas is increased. The increase of temperature is proportional to the current density $[I/\pi R^2, 2R]$ being the diameter of the tube and to the electric force X, which increases with the pressure.

In the experiments with large currents it is advisable to have the tubes closed so as to avoid large changes in the

density due to the increase of temperature of the gas.

13. The changes in the values of X and b with the current may be seen by the curves, fig. 4, which give the values of E obtained with currents from 3 to 18 milliamperes, the wavelength of the oscillations being 40 m. These experiments were made with the secondary circuit shown in fig. 2.

The four lower curves give the values of E for the tube 3.9 cm. internal diameter containing nitrogen at .26 mm.

pressure.

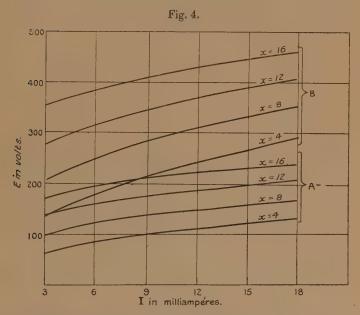
The curves correspond to the distances 4, 8, 12, and 16 cm. between the electrodes, and the value of X for any given current is $(E_1-E_2)/(x_1-x_2)$, where E_1 and E_2 are the electromotive forces for the distances x_1 and x_2 between the electrodes. In this case there is no appreciable change in X (from the value X=8.9 volts per cm.) in the b range of currents from 3 to 18 milliamp. The coefficient b, which is proportional to dE/dI, changes from the value b=14 with currents from 3 to 6 milliamp. to b=085 with currents from 12 to 18 milliamp.

The four upper curves (fig. 4) give the values of E for a tube 1.4 cm. internal diameter containing nitrogen at .53 mm. pressure. In this case the force X changes with the current, he values of $(E_1-E_2)/(x_1-x_2)$ being 17.7, 16.2, and 14.8 volts per cm. for the currents 3, 9, and 15 milliamp. respectively, and the coefficient b changes from the value b=.28 for currents from 3 to 9 milliamp. to b=.155 for

currents from 12 to 18 milliamp.

14. High frequency currents are maintained in helium and in neon with much smaller forces than those required to maintain similar currents in nitrogen. The electrodeless tubes used in the experiments with the monatomic gases were of quartz 30 cm. long and 4.2 cm. in diameter. With helium at 1 mm. pressure, the values of E for currents from 3 to 6 milliamp. are given by the formula:

$$E = 3.6 \times x + .036 \lambda I + 16,$$



Current-electromotive-force curves, $\lambda = 40$ metres. Curves A: nitrogen '26 mm. pressure, tube 3'9 cm. diameter. Curves B: nitrogen '53 mm. pressure, tube 1'4 cm. diameter.

and with neon at 1.06 mm. pressure E is given by the formula:

$$E = 2.5 \times x + .035 \lambda I + 12$$

which show that in the helium tube the force in the uniform glow was 3.6 volts per cm., and in the neon tube 2.5 volts per cm.

Thus the electrons acquire sufficient energy to ionize atoms of the gases when moving under these forces, so that in some of the collisions with the atoms the electrons lose a large proportion of their kinetic energy.

These results are in agreement with the experiments on the motion of electrons in gases which show that with the gases at 1 mm. pressure the average loss of energy of the electrons in collisions with atoms is small with forces of the order of 1 volt per cm., but the loss increases when the force increases. This is seen by the numbers in the following table, where δ represents the fraction of the energy of an electron which is lost in a collision with an atom of the gas. The values of δ are average values when the electrons move in a uniform field. The force X is given in volts per cm., the gases being at 1 mm. pressure.

Χ.	•5	1.	2.	4.
$\frac{\text{Helium}}{\delta \times 10^5}$	23	24	28.5	67.5
$\begin{cases} \text{Neon} \\ \delta \times 10^5 \end{cases}$	7	12	49	170

15. In order to compare the potentials required to maintain discharges with external and internal electrodes, tubes were made with internal electrodes of nickel sheet in the form of sleeves, 4 cm. long which fitted tightly into the tubes. Direct connexion to the electrodes was made through wires sealed in the glass, and the electromotive force between the internal sleeves was measured in the same way as the electromotive force between external sleeves.

Tubes of the same diameter containing nitrogen at the same pressure were also made up which were used with external electrodes.

The potentials required to start the discharges were determined with tubes containing nitrogen at various pressures from '005 mm. to 1 mm., also the minimum potentials required to maintain small currents with a uniform glow in the tubes.

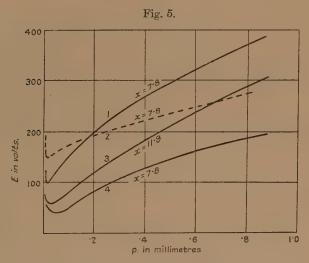
The results obtained with tubes 3.1 cm. in diameter for pressures from 0.4 mm. to 1 mm., are shown by the curves fig. 5. In this diagram the ordinates represent the potential E in volts, and the abscissæ the pressures p in millimetres, the wave-length of the oscillation being 40 m.

The potentials E required to start a discharge in a tube with external electrodes are given by curve 1 (fig. 5), and the corresponding potentials for the tube with internal electrodes are given by curve 2 (fig. 5), the electrodes being 7.8 cm. apart in each case. At pressures greater than 1.6 mm. the

electromotive force between the internal electrodes is smaller, but at lower pressures it is greater than the electromotive force between external electrodes.

16. The potential required to maintain a discharge is much less than the starting potential, and there is a minimum value of the potential that maintains a current with a uniform glow in the tube both with internal and external electrodes.

In general the glow is uniform along the whole length of the tube, and the intensity of the glow increases with the amplitude of the current, but with currents below a certain value the glow disappears from a space midway between the



Nitrogen: tubes 3.1 cm. internal diameter, $\lambda=40$ metres.

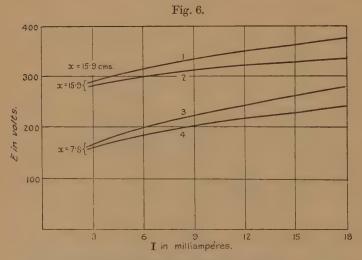
electrodes, and the luminosity is confined to the gas near the electrodes. When the force acting on the secondary circuit is reduced (by reducing the current in the generator), there is an abrupt diminution of the current in the tube, and an increase in the electromotive force between the electrodes, when the glow disappears from the middle part of the tube.

The minimum potential required to maintain a current with a uniform glow is easily determined, as the rate of change of the potential with the current is small, when the current approaches the value at which the glow ceases to be uniform along the tube.

The minimum potentials were determined for the two tubes 3.1 cm. in diameter, one with internal electrodes and the other with external electrodes 7.8 cm. apart, also for two

similar tubes with electrodes 11.9 cm. apart.

These potentials are given in terms of the pressure p by curves 3 and 4 (fig. 5). The minimum potentials are the same for tubes with internal and external electrodes for pressures greater than 04 mm. At this pressure, with the electrodes 7.8 cm. apart (curve 4), the potential required to maintain a small current is 39 volts; but it increases with the pressure, the minimum potential being 81 volts when the gas is at 2 mm. pressure and 189 volts at 8 mm. pressure.



Nitrogen pressure '53 mm., tubes 3.1 cm. in diameter, $\lambda = 40$.

At pressures less than '04 mm, the minimum potential increases as the pressure is diminished, and the potential is less for the tubes with external electrodes than for those with internal electrodes. For the tubes with electrodes 7.8 cm, apart the potential was 64 volts with external electrodes and 92 volts with internal electrodes when the nitrogen was at '01 mm, pressure.

17. The potential required to maintain large currents from 3 to 18 milliamp. were determined with the tubes 3·1 cm. in diameter containing nitrogen at ·53 mm. pressure, the wave-length of the oscillations being 40 m. The results of the experiments are given by the curves (fig. 6), where the

ordinates are the potentials in volts and the abscissæ the currents in milliamperes.

The potentials for currents in the tubes with external electrodes at distances 15.9 and 7.8 cm. apart are given by curves 1 and 3 respectively.

The corresponding potentials for currents in the tubes with internal electrodes at these distances apart are given by curves 2 and 4.

The difference between the ordinates of curves 1 and 3 is the same as the difference between the ordinates of curves 2 and 4, which shows that the force X in the uniform glow in the tubes with external electrodes is the same as the force in the tubes with internal electrodes. This force is approximately 14 volts per cm. with currents from 3 to 9 milliamp., but with larger currents the force diminishes as the current increases.

For a given current the potential E is greater with external electrodes than with internal electrodes. The difference is small, about 8 volts with a current of 3 milliamp., but it increases with the current, and with 18 milliamp. the difference is about 40 volts.

The difference between the values of E obtained with external and internal electrodes may be attributed to the fall of potential between the external electrodes and the inner surface of the tube, which is proportional to the current. Since the effect is negligible with small currents, the maintenance of the current cannot be due to any appreciable extent to electrons set free from the internal electrodes—unless electrons are set free at the same rate from the surface of the glass when external electrodes are used, which is very improbable.

18. Two tubes 3.1 cm. in diameter, with internal electrons in the form of sleeves, were used to determine the force in the positive column of a continuous current discharge in nitrogen at .53 mm. pressure. The current was maintained by a battery of small accumulators, which was connected through a high resistance [10^5 to 3×10^5 ohms] to the electrodes, and was adjusted by changing the potential of the battery or the resistance. With this arrangement the positive column became striated with currents of the order of 1 milliamp., but with smaller currents a uniform positive column was obtained. The electrodes were 16 cm. apart in one of the tubes and 7.5 cm. apart in the other, but with the same current in each tube the difference between the lengths of the positive columns (L_1-L_2) was not 8.5 cm. (as is

usually the case with plane electrodes). With currents of 0·15, 0·2, and 0·3 milliamp, the values of (L_1-L_2) were 8·5, 8·2, and 7·7 cm., and for these currents the values of V_1-V_2 were 112, 106, and 94 volts respectively, where V_1 was the electrometry force between the electrodes 16 cm. apart and

V₂ between the electrodes 7.5 cm. apart.

These numbers give the following values of the force $(V_1-V_2)/(L_1-L_2)$: 12·7, 12·9, and 12·2 volts per cm., which are the forces in the positive column. There is some uncertainty as to the accuracy of these measurements, since the values of (L_1-L_2) are not the same as the difference between the distances of a part of the electrodes in the two tubes. Further experiments are being made to check the accuracy of these determinations.

The average of the above forces, 12.6 volt per cm., may be taken as the force in the positive column of a continuous current discharge in a tube 3.1 cm. in diameter containing nitrogen at .53 mm. pressure. The root-mean-square value of the force in a high frequency discharge in a similar tube was X=13.7 volts per cm., so that the average intensity of the high frequency force $(2X \sqrt{2}/\pi)$ was 12.4 volts per cm., which is in good agreement with the value obtained for the force in a continuous current.

LXXII. On the Distribution of Charge in the Chlorine Ion in Rocksalt. By G. W. Brindley, Darbishire Research Fellow, and R. G. Wood, Manchester University*.

1. Introduction.

IN comparing the quantitative measurements of the intensities of X-ray reflexions from rocksalt and sylvine † with theory, a fairly close agreement was obtained between the experimental F curves ‡ for Na⁺, Cl⁻, and (K⁺+Cl⁻) and the theoretical curves calculated from the radial distribution of charge in these ions, corrections being made for the thermal vibrations of the ions and for zero-point energy.

* Communicated by Prof. W. L. Bragg, F.R.S.

† James and Firth, Proc. Roy Soc. A, vol. cxvii. p. 62 (1927); James and Brindley, Proc. Roy. Soc. A, vol. cxxi. p. 155 (1928).

[‡] F, the scattering factor for an atom, is a function of $(\sin\theta)/\lambda$ and may be defined as the ratio of the amplitude of the coherent radiation scattered from an atom in a state of rest to the amplitude scattered by a free electron according to the classical theory due to J. J. Thomson.

In each case, however, it was observed that for values of $(\sin\theta)/\lambda$ of the order of 0.4-0.5, θ being the glancing angle of incidence and λ the wave-length in Angström units of the X-radiation, the experimental points lay markedly lower than the theoretical curves. Since the form of the F curve depends directly upon the distribution of charge in the diffracting medium, it is of interest to inquire what variations of the charge distribution will account for the discrepancy between the observed and calculated F curves.

Certain assumptions have been made in comparing theoretical and experimental F curves. Firstly, it is generally considered that in rocksalt and sylvine the diffracting centres are ions. Measurements of the intensities of X-ray spectra, while consistent with this view, do not provide conclusive proof that the atoms in these crystals are ionized. Secondly, in order to compare experimental results with theory, it was necessary to assume that the radial distribution of charge in the ions in a crystal lattice was the same as that which would exist in the ions by themselves. This cannot be more than approximately true, owing to the overlapping of neighbouring ions and their mutual interaction. A third assumption was made, namely, that the ions could be considered spherically symmetrical. Though probably true to a first approximation for the inner electrons, this will not be true for the outer ones, since these are controlled least by the parent atom and are influenced most by neighbouring

Assuming still that the atoms are ionized, the question is asked how the charge-distribution in an ion in a lattice differs from that in a free ion. The charge-distribution in a free ion is determined by the field of the ion itself, but in an ion in a lattice the distribution is modified by the field due to neighbouring ions. This modification is partly a distortion of the spherical symmetry of the free ion, and partly an alteration in the radial distribution. Reasons will be given for supposing that the departure from spherical symmetry of an ion in a lattice of the rocksalt type is probably small, so that the modification due to neighbouring ions is mainly a change in the radial distribution.

The experimental evidence so far obtained gives little information about the symmetry of ions. If they were distorted it would be expected, in general, that the values of F for spectra of different type but of the same structure factor would lie on slightly different curves. No experimental evidence of this kind has yet been obtained. This tends to show that the charge-distribution in an ion approximates to

spherical symmetry, and that any departure from spherical symmetry is confined to the outer parts of the ions, the effect of which on the F curve is small. J. E. Lennard-Jones and H. J. Woods * have recently investigated theoretically the distribution of electrons in a two-dimensional "metal" and they find that surrounding each atom there is a circular distribution of electrons which is disturbed very little on the lines joining the centres of adjacent atoms, but that there is a certain distribution of charge in the spaces enclosed by four atoms. Such evidence as we have, therefore, tends to show that the lack of spherical symmetry is probably small, particularly for the inner "shells" of electrons.

It seems probable, therefore, that the difference between the experimental and theoretical curves may be due to the radial distribution of charge being altered by neighbouring ions. If we can neglect the lack of spherical symmetry, then the radial distribution of charge corresponding to the experimental F curve will be the distribution in an ion in a

attice.

There is, indeed, another possibility, namely, that the approximate method by which Hartree has determined the radial distribution of charge in a free ion may not be sufficiently accurate. Reasons will be given later to show that this is probably not the case.

2. Application of the Method of Fourier Analysis.

A. H. Compton \dagger has derived an expression by means of which the radial distribution of charge in an atom may be calculated from its F curve by a method of Fourier analysis. If U(r) be the electron density at a distance r from the centre of an atom, then

$$U(r) dr = \frac{8\pi r}{D^2} \sum_{1}^{\infty} n F_n \sin 2\pi n r / D \cdot dr. \qquad (1)$$

In this equation, F_n is the value of F for the nth order spectrum for planes whose spacing is D. The most direct method of obtaining the radial distributions corresponding to the experimental F curves would be to apply equation (1), giving D some arbitrary value. A difficulty arises due to the factor nF_n not converging sufficiently rapidly. The experimental F curves have to be extrapolated until F is inappreciable. In practice it is found that at the greatest

^{*} Proc. Roy. Soc. A, vol. cxx. p. 727 (1928). † X-Rays and Electrons, p. 151 et seq.

Distribution of Charge in the Chlorine Ion in Rocksalt. 619 angles at which F_n has been measured, nF_n is still large, and a big proportion of the terms in the series $\sum_{n=1}^{\infty} nF_n \sin 2\pi nr/D$ depends upon extrapolated values of F, the numerical value of many of them being large. In consequence, the final

of many of them being large. In consequence, the final result is influenced to a very large extent by the way in which the F curves are extrapolated. It is not practicable, therefore, to determine the U(r) curve by the direct application of Fourier analysis to the experimental F curve.

The following method, which was suggested by Prof. W. L. Bragg, eliminates the difficulties arising from the direct analysis. The difference between the experimental and theoretical F curves exists only for values of $(\sin \theta)/\lambda$ of the order of 0.4 to 0.5; for larger values of $(\sin \theta)/\lambda$ there is a very close agreement between the curves. Since F at large values of $(\sin \theta)/\lambda$ depends almost entirely on the inner electrons, for which the distribution of charge is known with most certainty and which are influenced least by neighbouring atoms, the error incurred by assuming the two curves identical for large values of $(\sin \theta)/\lambda$ will only be small. If then the Fourier analysis be applied to the difference, ΔF , between the two curves, the series $\sum_{n} \Delta F_n \sin 2\pi n r/D$ will converge very much more quickly than the series $\sum n \cdot F_n \sin 2\pi n r/D$, since ΔF_n becomes zero when F_n is still appreciable. This method eliminates the difficulties arising from the extrapolation of the experimental values of F, and the assumption which it involves is, on general grounds, justifiable.

3. Results.

The method has been applied to the results obtained by James and Miss Firth * for the chlorine ion, Cl^- , in rocksalt. The radial distribution of charge for this ion has been calculated by Hartree †, and the theoretical values of F for the Cl^- ion at rest are given in a joint paper by James, Waller, and Hartree ‡. In this last paper the values are also given of F for the Cl^- ion at rest, calculated from the experimental values of F at 86° Abs., by applying suitable corrections for thermal vibration and zero-point energy. The very small differences which occur between the theoretical and experimental values of F at values of $(\sin \theta)/\lambda$ greater than 1.0,

^{*} Proc. Roy. Soc. A, vol. exvii. p. 62 (1927).

[†] Camb. Phil. Soc. Proc. vol. xxiv. pp. 89, 111 (1928).

[†] Proc. Roy. Soc. A, vol. exviii. p. 334 (1928).

which are only of the order of 0.15 of an electron, will be neglected in the present paper, since differences so small may well be due to the difficulty in measuring very weak

spectra.

The analysis has been carried out for two values of the arbitrary spacing D, 5 Å.U. and 10 Å.U. In Table I. values are given of F_n , calculated from the Hartree-Schrödinger radial distribution of charge and of F_n , calculated from the experimental results of James and Miss Firth, for a series of values of n, D being 10 Å.U.

Table I. $D = 10.0 \text{ Å.U.} \quad (\sin \theta) / \lambda = n/2 D = 0.05 \text{ n.}$

n.	$(\sin \theta)/\lambda$.	$F_{n'}$ theoretical.	\mathbf{F}_n experimental.	$\mathbf{F}_n' - \mathbf{F}_n^{\omega}$.	n , $\Delta \mathbf{F}_n$.
1	•05	17:11	17:11		_
2	•10	15.23	15.23		
3	·15	13.19	13.19		-
4	•20	11.50	11.50		
5	·2 5	10.23	10.15	•08	•40
6	•30	9.30	9.02	•28	1.68
7	•35	8.60	8.32	.28	1.96
8	•40	8.06	7.70	· 3 6	2 ·88
9	•45	7.62	7.20	•42	3.78
10	•50	7.23	6.78	•45	4.50
11	•55	6.84	6.37	•47	5.17
12	·60	6.49	6.03	·46	5.52
13	•65	6.12	5.75	•37	4.81
14	•70	5.77	5.45	•32	4.48
15	·75	5.41	5.16	•25	3.75
16	•80	5.06	4.89	·17	2.72
17	*85	4.74	4.62	·12	2.04
18	•90	4.41	4.35	•06	1.08
19	•95	4.10	4.10	-	

In Table II. the results are given of the Fourier analyses. If $\Delta U(r)$ be the difference between the values of U(r) corresponding to $F_{n'}$ and F_{n} , then

$$\Delta \mathbf{U}(r)dr = \frac{8\pi r_n^{\infty}}{\mathbf{D}^2} \mathbf{n} \cdot \Delta \mathbf{F}_n \sin 2\pi n r/\mathbf{D} \cdot dr.$$

On the left-hand side of Table II. values are given of $\Delta U(r)$ for a series of values of r/D, the latter being chosen to facilitate the calculation of the sine terms. On the right-hand side of Table II. are tabulated values of $\Delta U(r)$ for a series of values of the radius r, these values of $\Delta U(r)$ being obtained by graphical interpolation from the results given on the left-hand side of the Table.

TABLE II.

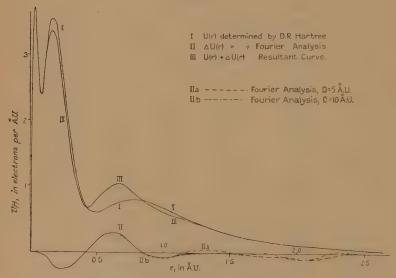
r/D ,	$\mathbf{U}(r)$ in electrons/A.U.		m in	$\mathrm{U}(r)$ in electrons/A.U.		
	D=5.0 Å.U.	D=10 Å.U.	r in Å.U.	D = 5.0 Å.U.	D=10 Å.U.	
.025	-1.13	-2.40	0.1	-0.62	-0.26	
.050	-2 ·45	+1.91	0.2	-2.35	-2.27	
.075	-1.05	+1.34	0.3	-2·28	-2.21	
·100	+1.85	-0.91	0.4	-0.47	-0.30	
•125	+2.95	+0.17	0.5	+1.85	+1.91	
·150	+1.51	-0.16	0.6	+2.92	+2.92	
.175	-0.26	+0.01	0.7	+2.42	+2.33	
·2 00	-0.72	-0.84	0.8	+0.60	+0.45	
•225	-0.16	+0.19	0.9	-0.50	-0.60	
.250	+0.36	-0.09	1.0	-0.72	-0.92	
.275	+0.47	+0.55	1.1	-0.37	-0.71	
.300	+0.12	+0.02	1.2	+0.21	-0.05	
·325	-0.29	+1.01	1.4	+0.42	+0.50	
·350	-0.51	-0.90	1.6	-0.22	-0.36	
.375	-0.56	-0.59	1.8	-0·52	+0.11	
·400	-0.76	-0.57	2.0	0.78	-0.84	
•425	-0.83	+0.01	2.2	0.59	-0.37	
•450	-0.38	+0.23	2.4	+0.08	+0.48	
·475	+0.04	+0.81				
•500	0	0				

There is a close similarity between the two analyses for values of r less than 1.0 Å.U., but, as r increases, the discrepancy between the two analyses also increases. To some extent this is due to $\Delta U(r)$ being proportional to r. But this is not the sole cause. The arbitrary spacing D must be chosen greater than twice the radius of the atom, for when D is equal to twice the radius, the summation $\Sigma nF_n \sin 2\pi nr/D$ is zero. In taking D=5.0 Å.U., it is assumed that the charge lying outside a radius of 2.5 Å.U.

is negligible. This is only approximately true. It is probable, therefore, that the discrepancy between the two analyses is partly due to there being an appreciable amount of charge at distances from the nucleus greater than D/2. We have assumed that the oscillations of the $\Delta U(r)$ curve about the r-axis for r>1.0 Å.U. arise mainly, if not entirely, from the method of analysis. From r=1.0 Å.U. the $\Delta U(r)$ curve has been drawn to zero at r=1.5 Å.U. in such a

manner that $\int_0^\infty \Delta U(r) dr = 0$.

Fig. 1.



Distribution of Charge in Chlorine Ion, Cl-.

In fig. 1, curve I is the U(r) curve for the Cl^- ion calculated by Hartree. Curve II is the $\Delta U(r)$ curve which has been used to determine the $[U(r)+\Delta U(r)]$ curve—curve III in fig. 1. The actual results of the Fourier analyses, where they differ appreciably from curve II, are shown by the broken curves II a and II b, which correspond respectively with D=5.0 Å.U. and D=10 A.U.

The difference between the U(r) curve calculated by Hartree, and the curve corresponding to the experimental F curve, is greater than can be attributed to the approximate nature of Hartree's method. Gaunt, in a recent paper *,

^{*} Proc. Camb. Phil. Soc. vol. xxiv. p. 328 (1928).

has discussed the validity of Hartree's assumption regarding the self-consistent field, and he shows that when small corrections are made for the spin of the electron and relativity correction, the method is legitimate. Moreover, the numerical values of the ionization potentials calculated from Hartree's constant ϵ agree closely—in some cases to within 1 per cent.—with values obtained experimentally. It seems unlikely then that the U(r) curve determined by Hartree can be so much in error as the difference $\Delta \mathrm{U}(r)$ between the two U(r) curves in fig. 1.

It seems possible, therefore, that the difference between the theoretical and experimental values of F arises because the radial distribution of charge in a Cl⁻ ion in a rocksalt lattice differs from the distribution in a single Cl- ion. If the results can be interpreted in this way, then. from fig. 1, it is seen that the maximum of the charge distribution of the M electrons in the Cl- ion in rocksalt is sharper and occurs at a smaller radius than in the free chlorine ion.

4. Summary.

It has been found that the experimental and theoretical F curves for the chlorine ion, Cl-, differ by an amount greater than can be attributed to experimental error for

values of $(\sin \theta)/\lambda$ of the order of 0.4-0.5.

Reasons have been given for supposing that the lack of spherical symmetry of an ion in a lattice is small and that any departure from spherical symmetry is confined mainly to the outer part of the ion, which will have little effect on the F curve. We have therefore assumed that the difference between the experimental and theoretical F curves is due to a change in the radial distribution.

Applying the method of radial Fourier analysis to the difference ΔF between the two curves, the difference between the charge distribution in a free chlorine ion, as determined by Hartree, and in a chlorine ion in a rocksalt lattice has been calculated. Hence the charge distribution

in a chlorine ion in rocksalt has been obtained.

In conclusion, we wish to thank Prof. W. L. Bragg, F.R.S., and Mr. R. W. James for suggesting this investigation and for their interest. Thanks are also due to the Department of Scientific and Industrial Research for a grant held by one of us.

Manchester University, Dec. 19, 1928.

LXXIII. Notes on Surface-tension. By Alfred W. Porter, D.Sc., F.R.S., F.Inst.P., Emeritus Professor of Physics in the University of London *.

I. The Rise of Liquid in a Capillary Tube.

THE problem of the rise of liquid in a capillary tube received its last and most complete treatment by the late Lord Rayleigh in 1915 (Proc. Roy. Soc. A, xcii. p. 184, 1915). The differential equation that must be exactly satisfied is

$$\frac{d\sin\theta}{dx} + \frac{\sin\theta}{x} = \frac{1}{\beta^2}(h+y), \quad . \quad . \quad (1)$$

where y and x are the rectangular coordinates of the meniscus reckoned from the vertex, θ the inclination to the horizontal, and

$$\beta^2 = \text{surface-tension} \div (g \times \text{density}) = \sigma/(g\rho).$$

To a first approximation (i.e. neglecting y) the meniscus is a circle of radius a, from which y can be calculated and inserted in the equation, and thus further approximations can be obtained.

Putting

$$y = a - (a^2 - x^2)^{\frac{1}{2}} + u,$$

[or, as we may abbreviate it,

where

$$y = a - w + u,$$

 $w = (a^2 - x^2)^{\frac{1}{2}},$

whence, when needed,

$$x dx = -w dw,$$

$$\mathbf{u} = \left\{ \mathbf{K} \frac{a^2}{w} + \frac{a^3}{3\tilde{\beta}^2} \log (a + w) \right\}$$

$$+\frac{a^{5}}{6\beta^{4}}\left[-2\log\left(a+w\right)+\frac{w}{a}-1+\frac{a}{2w}+\frac{a}{w}\log\left(a+w\right)\right]\Big\}$$

+ constant of integration. (2)

The symbol K represents a constant obtained in the course of a previous integration.

Following the assumption which Poisson made, that u must never be allowed to attain an infinite value, the factor

^{*} Communicated by the Author.

of the terms containing w in the denominator only must vanish, because w becomes zero when $x^2=a^2$. Hence

$$u = \frac{a^2}{3\beta^2} \left(1 - \frac{a^2}{\beta^2} \right) \log \frac{a+w}{a} + \frac{a^5}{6\beta^4} \left[\frac{w-a}{a} + \frac{a}{w} \log (a+w) \right]$$
+ constant of integration;

and finally, the integration of (1) gives

$$1 = \frac{ha}{2\beta^2} \left[1 + \frac{a}{3h} - 0.1288 \frac{a^2}{h^2} + 0.1312 \frac{a^3}{h^3} \right], \quad . \quad (3)$$

as the equation by which β^2 , and therefore the surfacetension, can be calculated from experimental values of h in a tube of radius a.

The object of the present note is to discuss the mode in which the constant K is determined. Poisson and Rayleigh seem to have been guided to their mode of determining it by the fact that they saw no other way. Concerning the choice of K, Rayleigh remarks, "it may appear at first sight as though we might take it almost at pleasure."

The choice actually made is somewhat arbitrary, because, whatever the fact may be, there is no prima facie reason for excluding all infinite values of y (i. e. of u). The only obvious restriction that must be attended to is that u must never attain such values as would make infinite the total weight of the liquid raised, for this must equal $2\pi\sigma \cdot a$. Now a term in the expression for y varying inversely as w is therefore permissible, because its contribution to the total weight raised

is proportional to $\int_a^0 \frac{1}{w} w \, dw$, i. e. to $\left[\sqrt{a^2 - x^2}\right]_0^a$, which is finite.

In reality, we are not without a guide in the choice of the coefficient. It appears to have been overlooked completely that the value of K is fixed by the known curvature at the vertex. Since the total curvature there is h/β^2 and the two curvatures are equal, each of them must equal $h/2\beta^2$. The curvature can be found by twice differentiating (2), and thence it is found that, necessarily,

$$K = \frac{ha}{2\beta^2} - 1 + \frac{1}{6} \frac{a^2}{\beta^2} - \frac{a^4}{6\beta^4} \log 2a.$$

However, when this value is inserted in (2) it is found, finally, that no change is required in the equation (3) up to the order of terms there included.

Rayleigh's own formula for y gives the curvature at the vertex as being

 $\frac{1}{a} - \frac{a}{6\beta^2} + \frac{a^3}{6\beta^4} (\log 2 - \frac{1}{2}) ;$

this can be shown from (3) to be identical with $h/(2\beta^2)$ up to the order given. Hence his assumption appears to be true in fact, although it was introduced without having any safe basis.

The question whether infinite values of y are permissible or not can be discussed on other lines. At the vertex (equation (1)) the two curvatures are equal, and equal to $h/(2\beta^2)$. At the tube wall, the second curvature is 1/a; so that, if Y is the value of y there, the first curvature becomes

$$C_1 = \frac{1}{\beta^2}(h + Y) - \frac{1}{\alpha}.$$

To require an infinite value of Y would require, therefore, an infinite curvature at the wall. These demands would appear to be incompatible for the kind of curve with which we have to deal.

II. The Weight of Drops from Tubes of Various Diameters.

This question has also been discussed by Lord Rayleigh (Phil. Mag. xlviii. pp. 321-337, 1899), and he deduces the dimensional equation

$$\mathbf{M} = \frac{\sigma d}{g} f \left(\frac{\sigma}{g \rho d^2} \right) \cdot$$

On testing this equation by experiment, he finds that wide ranges of the term in the brackets give fairly constant values for $Mg/(\sigma d)$. Since we have no reason to expect that the diameter, D, of the drops bears a constant ratio to the diameter, d, of the tube, a function of a/D will also enter into the equation. The character of the experimental results obtained can be best understood if the variable in the brackets is changed by multiplying it by the zero-dimensional quantity $Mg/(\sigma d)$ or $\rho D^3 g/(\sigma d)$. Including both of these changes,

$$Mg/(\sigma d) = f \left[\frac{\sigma}{g\rho d^2} \frac{\rho D^3 g}{\sigma d}, \frac{d}{D} \right]$$
$$= \phi \left[\frac{d}{D} \right],$$

so that everything disappears on the right hand except a function of the ratio of the characteristic dimensions of

tube and drop. Experiment shows that the left hand has a minimum value for a particular external diameter of the tube; hence no simple-power law for the function ϕ will suit.

Treating the problem as a static one, the above seems to give all that can be obtained by this method. Since the problem in the case of falling drops is partly a kinetic one, the question of viscosity arises. Determining in the usual way suitable numerics as independent variables, the equation becomes

$$\frac{Mg}{\sigma d} = f_2 \left[\frac{g\rho^2 d^3}{\mu^2}, \frac{d}{D} \right].$$

Now, for a given tube the weight of a drop of olive oil is not conspicuously different from that of a water one, although its kinematic viscosity (μ/ρ) is a hundred times as great. We may therefore conclude that the viscosity has very little influence in practice.

III. The Vanishing of Surface-tension near the Critical Point.

Professor Callendar's recent experiments on steam (Proc. Roy. Soc. A, Sept. 1928) appear to show that the meniscus between the liquid and gaseous phases of steam disappears some six degrees below the critical point, the two temperatures being about 374° C. and 380° C. respectively. At the temperature of vanishing there is a difference of density (ρ steam=0.6 ρ water). Measurements have been made for water to nearly 380° C.; it was not found possible to make similar measurements for steam beyond 374° C., but the part of the curve obtained extrapolates without difficulty to meet the liquid curve in the neighbourhood of 380° C.

In the theory of capillarity, as worked out by Laplace, vanishing of the surface-tension should take place only when the densities of the two phases are the same. The object of this note is to point out that this is found not to be so when an attempt is made to get a better approximation by recognizing that the centres of molecules cannot approach nearer than their diameter. Laplace took the whole substance as a continuum, and certain of his integrations are effected between zero and infinity instead of between a distance comparable with the molecular diameter (s) and infinity. The Dutch school has made the change (which is by no means a slight one, because the range from 0 to s is infinitely important compared with the remainder for forces taken as

varying as an inverse high power of the distance between centres).

Assuming the inverse nth power of force between molecules, the surface-tension is $\rho^2 F$, where F is a function which might plausibly be the same even for different kinds of

matter; but on the modified theory F may be written $\frac{f}{s^{n-5}}$

where n is the inverse-power law of force and the surface-tension between any two media becomes

$$\sigma_{12} = \rho_1^2 \mathbf{F}_1 + \rho_2^2 \mathbf{F}_2 - 2\rho_1 \rho_2 \mathbf{F}_{12},$$

where

$$F_1 = f/s_1^{n-5}$$
, $F_2 = f/s_2^{n-5}$, $F_{12} = f/s_{12}^{n-5}$.

On Laplace's theory, where the values of F are all the same,

$$\sigma_{12}=(\rho_1-\rho_2)^2\mathrm{F},$$

so that the tension vanishes only when the densities ρ_1 and ρ_2 are equal. Another possibility arises in the more general case. To take as simple an illustration as possible, let us assume that n=7, and that s_{12} can be taken as the geometric mean of s_1 and s_2 ; then

$$\sigma_{12} = f \left\{ \frac{\rho_2}{s_2} - \frac{\rho_2}{s_1} \right\}^{\scriptscriptstyle \parallel},$$

and it appears that

- (1) if the molecules are alike when the densities are alike, σ_{12} then equals zero;
- (2) the relative tension will also be zero if $\frac{\rho_1}{s_1} = \frac{\rho_2}{s_2}$.

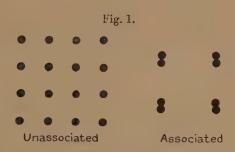
For example, let us suppose as for steam that $\rho_2 = 6 \rho_1$, then at that point the surface-tension is zero (and the meniscus becomes plane), provided that $s_2 = 6 s_1$. Such a possibility may easily arise if the vapour associates on condensing into

a liquid, as the following diagram shows (fig. 1):

Here, in the second phase the density is greater, and yet the distance between neighbouring molecules is also greater. If the temperature is increased until the mean distances happen to be in the right proportion, the surface-tension vanishes, in spite of the inequality in the densities. Moreover, if at some higher temperature the liquid dissociates so that the molecules become of the same complexity as in the vapour state, the surface-tension will again vanish when the densities become equal. The curve of surface-tension against temperature will in

such cases become somewhat as shown in fig. 2.

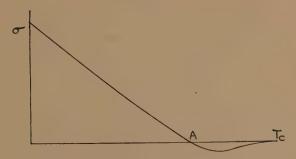
This result is in accordance with the measurements of the latent heat of steam; it is at the higher of these two temperatures that the latent heat vanishes. Now E. T. Whittaker has shown that there is a close parallelism



between the internal latent heat and the surface energy, u. Thermodynamically the connexion between u and σ is

$$u = \sigma - T \frac{\partial \sigma}{\partial T}.$$

Fig. 2.



Since u may be expected to vanish when L_{int} vanishes (i. e. at T_c), then $\frac{d\sigma}{dT} = 0$ at that point. At the lower point,

A, σ vanishes, but L_{int} , and therefore u, do not vanish. This indicates that at A, $d\sigma/dT$ is not zero. The diagram shows that it has a negative value there, and therefore u is positive as we would expect.

When the problem is studied from the point of view of Eötvös's law, it is recognized that his straight line extrapolated does not pass through the critical point, but some 5 to 7 degrees below it. This fact is usually accommodated to thermodynamical requirements by representing the surface-tension by an equation such as $\sigma = A(T_c - T)^n$, where n is about 1.2. But in view of the considerations brought forward in this note, the question arises whether, after all, a curve such as is shown in fig. 2 represents in reality a usual instead of an exceptional case. It must be mentioned that the case of steam is not the only one that shows the two special temperatures. Even Waterston, making experiments in closed tubes after the manner of Cagniard de la Tour, found that the cup-shaped meniscus caused by capillarity ceased at a temperature considerably under the point of transition (Phil. Mag. Jan. 1858). In alcohol the surface became quite flat at 422° F., while the transition occurred at 540° F. The relative densities of vapour and liquid at the lower point were 0.222 and 0.480. He quotes Wolf (Ann. de Chimie, xlix) as showing that for ether the temperatures are 190° C. and 200° C. Both Wolf and Waterston observed a convexity of the surface between these temperatures. This, of course, does not mean that the interfacial tension becomes negative, but it is consistent with such a value.

It is not to be supposed that the suggestion made here is sufficient to account for the whole of the quantitative behaviour of steam in contact with water. If there are indeed two or more types of molecules present (associated and dissociated), we have to deal with at least a binary mixture, and any of the complications investigated by van der Waals, Rooseboom, Verschaffelt (H2+CO2), Caubet (CO₂+SO₂), Smits (anthrachinone and ether), and others may arise. The theory has only been worked out for binary mixtures of distinct components. In the case of steam, according to Callendar we are dealing with two states of the same component, i.e. a mixture in which the two types of molecule can transform into one another. I am not aware that the case of a dissociating mixture has yet been dealt with theoretically. One would anticipate it to be a problem of extraordinary difficulty.

87 Parliament Hill Mansions, N.W. 5. January 25, 1929.

LXXIV. On the Separation of Isotopes. To the Editors of the Philosophical Mayazine.

GENTLEMEN,—

TE should be glad if you would allow us to make a short comment on the paper of Harkins and Mortimer published in the October issue of the Philosophical Magazine. This paper deals with the problem of the Separation of Isotopes, and contains also a detailed account of the history of the subject. Early attempts were made to separate the radioactive isotopes by diffusion in liquids, and especially to separate uranium 1 from uranium 2; but these attempts failed. To explain this failure, it was pointed out that the rate of diffusion of ions in liquids is practically independent of the mass, and is inversely proportional to the radius; as the radius of the isotopes is practically identical, hardly any separation could be expected by this method. Even at this early date it was suggested that in a gaseous medium diffusion is bound to lead to a separation of isotopes. Such an attempt was, in fact, made by Aston quite independently of the above considerations. As announced at the meeting of the British Association at Birmingham in 1913, he was successful in obtaining a partial separation of the isotopes of neon by diffusion.

In 1920 the present writers attempted the separation of the isotopes of mercury; they based their method on an entirely new principle-namely, that the rate of evaporation of isotopes must be inversely proportional to the square root of their atomic weights. This method allows an exact calculation of the effect to be expected, and in our experiments we obtained a long series of results which were in the best agreement with the theoretical values. The partial separation of the isotopes was followed by density measurements, and the reality of the phenomenon was proved by the fact that the various samples retained their density values unchanged on ordinary distillation. Furthermore, a measurement of the electrical conductivity of the fractions obtained was carried out by Jaeger and Steinwehr, who found the conductivity of the various samples to be identical within about one part in a million, and from this it was concluded that no impurities were present. Finally, Hönigschmid determined the a omic weight of the fractions obtained, and found that these agreed well with the values

calculated from density differences.

In view of these facts, we have some difficulty in understanding the remarks of Harkins and Mortimer, in the paper quoted above, that we gave no evidence that our mercury

was pure.

Aston's diffusion method and our method of ideal distillation were used by Harkins and his collaborators §, by Egerton, and by others. By improving the technique of the experiments and by laborious work, Harkins and Mortimer succeeded in obtaining mercury fractions the atomic weight of which differed by 0.2 of a unit. The present writers seven years ago obtained mercury fractions the atomic weight of which differed by 0.1 unit. This illustrates clearly the fact that, while methods based on the difference of molecular velocities are very useful in obtaining a partial separation, they are quite incapable of effecting a full separation.

Yours faithfully, J. N. Brönsten and G. Hevesy.

LXXV. The Corona Discharge in Neon. To the Editors of the Philosophical Magazine.

GENTLEMEN,-

In the April issue of the Phil. Mag. (vol. v. No. 30, p. 721) Mr. Huxley published an article on the corona discharge in helium and neon. He found that, at higher pressures, the negative discharge (wire cathode) started at a higher potential difference than the positive; this effect was especially very pronounced in neon. In the opinion of Mr. Huxley this result forms a strong argument against the theory that in corona discharges the electrons should be liberated mainly from the cathode by the action of the positive ions.

When this article was published, I had done some work on sparking potentials between coaxial cylinders in argon and neon. These experiments showed that the effect mentioned previously manifested itself only in case the gas was not quite pure. In order to perform measurements directly comparable with those of Mr. Huxley, a tube was designed with an electrode system of about the same dimensions, but of a more simple construction. The outer cylinder was made of chromium iron (diameter 4.6 cm.) which was sealed to

the glass; the inner cylinder consisted of a nickel wire (diameter 0.175 cm.) protected on both sides by glass insulation. In a side-tube another electrode system was mounted in order to determine also the sparking potential between parallel plates (distance about 1 cm.) for the same gas. After pumping the whole apparatus at high temperature in the usual way, 40 mm. neon gas was admitted, which then was freed from traces of impurities by a glow discharge. From little side-tubes small amounts of argon could be admitted.

The results for the starting potentials of the positive, the negative discharge, and the discharge between the parallel plates are given in the following table:—

Gas.	Wire positive.	Wire neg.		
		wire neg.	Parallel pl.	
Neon	616 V.	555 V.	512 V.	
Neon+0.0001 per cent. Ar	490 √ .	542 V.	477 V.	
Neon+0.0004 per cent. Ar	309 V.	465 ∇ .	408 V.	
Neon+0.0011 per cent. Ar	206 V.	400 ∇.	334 V.	

Now, it was shown previously (F. M. Penning, Naturwissensch. xv. p. 818, 1927; Zs. f. Phys. xlvi. p. 535, 1928; Physica, viii. p. 13, 1928; Versl. Kon. Ak. v. Wet. Amsterdam, xxxvii, p. 308, 1928) that the effect of very small traces of impurities on the sparking potential of neon is brought about by the circumstance that the impurities are ionized by excited (and especially by metastable) neon atoms. As the laws for the generation etc. of these metastable atoms may be different for the positive and negative discharge, there may be also a large difference in the sparking potentials in these two cases. At any rate, according to the present experiments the effect to which Mr. Huxley refers does not exist in pure neon; the starting potential is there even somewhat higher for the positive discharge than for the negative. So the experiments of Mr. Huxley give no material against the theory that electrons should be set free from the cathode by the action of the positive ions.

Natuurkundig Laboratorium der N. V. Philips' Glocilampenfabrieken, Eindhoven (Holland). December 1928. Yours faithfully, F. M. PENNING.

LXXVI. Notices respecting New Books.

The Evolution of the Igneous Rocks. By N. L. BOWEN. (Princeton University Press. 1928. Price \$5.00.)

'HIS book embodies a course of lectures delivered to advanced students, and-as defined in the preface-is designed to interpret the outstanding facts of igneous-rock series as the result of fractional crystallization. The use of the term "evolution" is intended to designate a process of derivation of rocks from a common source.

The subject of fractional crystallization is taken up in the sixth chapter. We find, as we expect, that the mother substance is essentially basalt. Later, Professor Bowen defines such mother substances as not far from saturation with pyroxene and plagioclase, and he refers to the Deccan traps and plateau basalts in general as representing the parent magma. In this he is in accord with other writers—notably Daly. Here, also, he defines the ophitic structure of basalts as probably due to simultaneous formation of two minerals, one of which (the felspar) possesses the greater tendency towards idiomorphism. This important subject is developed with the same clearness and attention to detail which characterizes this book.

Further on Professor Bowen expresses his adherence to Daly's view that basaltic magma acts as heat-bringer, and enters into igneous rock economy on a different basis from all other magmas a thesis fully developed by Daly in his 'Igneous Rocks and their Origin,' who claims that the facts of volcanic geology seem to cooperate with the facts of plutonic geology in showing that the essential process in igneous action on our planet is the rise of basaltic magma from the universal substratum along abyssal fissures.

Such great generalizations clearly define the importance of petrological science as contributory to our knowledge of the surface history of the earth. Professor Bowen himself has done much to advance views as to the fundamental character of basaltic magma. The view that anorthosites and peridotites arise out of crystal accumulations in basaltic magma is now generally accepted. Evidence that these important rock-materials were ever in the liquid form has not been forthcoming. This important generalization is largely referable to the earlier work of Professor Bowen.

On the subject of the origin of many granitic magmas little is as yet known. The primitive origin of granite on a great scale as contributing to the earth's acid shell is generally, of necessity,

accepted.

Professor Bowen refers to the question of the melting points of basalt and of granite. Doubtless both time-element and dynamic-element enter into this difficult question. The present writer, with a view to bringing in the time-element, made experiments many years ago on rock-forming minerals, an account of which appears in the Compte-rendu of the International Geological Congress of 1900. They are, on the whole, opposed to the recent results of Shepherd and Craig, to which he refers. And, on the large scale, the present writer has not arrived at their result. A cube of fine-grained granite, floated for some hours in fluid basalt, when ultimately sectioned showed its external form to be unaltered; the quartz still vigorously double-refractive—the felspar more or less vitrified; the muscovite altered to the extent of liberation of its water of crystallization; the biotite slagged. Moreover, fluid basalt may be retained in a silica crucible for an indefinite time without any apparent effect upon the crucible. The data cited by Clarke in his 'Data of Geochemistry,' 5th edition, are to the same effect.

Passing over many matters of interest, we arrive at a concluding chapter on "Petrogenesis and the Physics of the Earth," which is more comprehensive than any statement upon this subject offered by American or British petrologists within recent years,

so far as the present writer is aware.

Professor Bowen considers that geologic evidence respecting the outer layers of the earth favours the larger estimates arising out of seismic observations, *i.e.* a granitic layer about 25 kms. in thickness, gradually changing through intermediate to basic rocks, which give place abruptly to an ultrabasic layer at a depth of about 60 kms. [Recently Conrad has shown that the thickness of the granitic layer appears to be about 40 kms. (Gerlands

Beiträge zur Geophysik, xx. p. 275).]

Professor Bowen objects decisively to the assumption that . below a certain level (10 to 20 kms. of average granite) there must be no radioactive substances, or that their quantity must fall off so rapidly as to become negligible. "This result is not Below the granite layer there must, in any ratio al acceptable. scheme of things, be intermediate and basic rocks whose radioactive content is far from negligible. Below these again the ultrabasic rocks must be regarded as having the lower, but still important, quantity of radioactive substances usually found in ultrabasic rocks." He refers to certain of the earlier objections which have been raised to the view that thermal cycles are responsible for the discharge of deep-seated accumulation of heat, but adds: "In spite of the various objections, it is not improbable that an hypothesis involving periodic discharge of heat in great quantity will ultimately be preferred."

Finally he discusses the nature of the deep-seated ultrabasic layer, favouring, on the whole, the view that it is peridotic in character, but admitting the possibility that it may be eclogite,

as suggested by Holmes and the present writer.

It is impossible, in the available space, to do adequate justice to Professor Bowen's comprehensive book. It is lucidly written and illustrated with many excellent diagrams. It will be valuable to

the advanced student, and will certainly find a place in the library of all who are interested in the science of the rocks or in geophysics generally.

J. J.

Annual Tables of Constants and Numerical Data.—Chemical, Physical, Biological, and Technological. Vol. VI. (1923-24). [Part I., pp. xxxiv+680; Part II., pp. xxix+995.] (Paris: Gauthier-Villars et Cie. 1927-28. Price: paper covers, 530 f.; bound, 610 f.)

THESE volumes are the latest additions to the well-known series of Annual Tables of Constants and Numerical Data, which are published under the patronage of the Union of Pure and Applied Chemistry by an International Publishing Committee, subsidised by numerous governments, academies, scientific societies, industrial societies, commercial firms, and private individuals. They contain all the numerical data that have appeared in the scientific publications throughout the whole world for the years 1923-24.

As compared with previous volumes, several additions and improvements have been made. The additions include chapters on Photography, Radioelectricity, and Geophysics. In the chapter on Engineering and Metallurgy the whole of the text is given in both French and English, and it is intended that this improvement will be made throughout the whole of succeeding yolumes. Another improvement is the addition of detailed indexes to several of the chapters.

Volume VII., covering the years 1925-26, will be published

during the present year.

Certain of the chapters, such as Spectroscopy, Photography, Radio-electricity, Electricity, etc., are published separately, which is a convenience for those who do not require the whole volume. Specimen pages of these and any further information can be obtained from M. Ch. Marie, General Secretary of the International Publishing Committee, 9 rue de Bagneaux, Paris (VI.).

The Principles of Thermodynamics. By G. BIRTWISTLE. Second Edition. [Pp.ix+168.] (Cambridge; At the University Press. 1927. Price 7s. 6d. net.)

The second edition of Mr. Birtwistle's small volume on the Principles of Thermodynamics is essentially the same as the first edition, with the exception that a chapter on Nerust's heat theorem has been added. Minor revisions have also been made where required. That so few changes have been found necessary for the second edition indicates that the concise, well-balanced treatment of the subject is well adapted to the needs of the student who is desirous of obtaining a logical understanding of the principles of thermodynamics. The volume can be strongly recommended as a concise introduction to the subject.

Lehrbuch der Physikalischen Chemie, von Dr. Karl Jellinek. Fünf Bäude. II. Bd. Bogen 1-17. [Pp. 272 mit 241 Figuren.] (Stuttgart: Ferdinand Enke, 1928. Preis M.21; geheftet.)

THE first volume of the new edition of Dr. Jellinek's 'Lehrbuch' was reviewed recently in these pages. Each of the remaining volumes will be issued in three sections, at intervals, at the rate of one volume per year during the years 1928 to 1931. The first section of the second volume has already appeared: it deals with the properties of matter in the solid state, both isotropic and anistropic bodies being considered. A large portion is devoted to the crystalline state, the classification of crystals, crystal properties, axes, angles and planes of symmetry, including an account of the information derived from the diffaction of X-rays by crystals.

The elastic and thermal properties of solid bodies are considered in detail, including sections on specific heat, calorimetry, thermal conductivity, melting and solidification, and the effect of pressure on melting point, sublimation and the quantum theory of specific

heats.

Theoretical considerations are throughout carefully explained and experimental methods are considered in great detail. Numerous tables of observational data increase the value of the work for reference purposes. The volume is one which should find a place in every chemical or physical library.

Lehrbuch der Physikalischen Chemie, von Dr. Karl Jellinek.
Zweite Ausgabe. Bd. II., Bogen 18-35. [Pp. 273-560; Preis M.24]; Bd. II., Bogen 36-58 u. Titelbogen. [Pp. 561-924+xv; Preis M.32]. Bd. II., Vollständig. [Pp. xv+924. mit 384 Figuren und 148 Tabellen. Preis: gch. M.88; geb. M.92.]
(Stuttgart: Ferdinand Enke. 1928.)

The two sections of Bd. II. of Jellinek's 'Lehrbuch' complete the second volume. The remaining three volumes will be published in the years 1929, 1930, and 1931 respectively. The parts under review deal first with the application of kinetic and thermodynamical considerations to specific heat, entropy, melting and solidification, heat conductivity, surface energy of crystals, &c. The properties of gaseous and fluid mixtures are then considered, including diffusion of gases, osmotic pressure and its measurement, boiling and freezing points, &c. The final section deals with crystalline mixtures.

Both theoretical and practical aspects of the subjects dealt with are treated very fully. The mathematical factors are clearly expounded and can be followed without difficulty by those with but an elementary knowledge of mathematics. The text is illustrated by numerous figures and tables of numerical and experimental data. On account of its completeness and the care exercised in its compilation, the volume will serve not only as a

text-book but also as a valuable work of reference.

Statistical Methods for Research Workers. By R. A. FISHER, Sc.D. (Biological Monographs and Manuals, No. V.) Second Edition. Pp. xiii+269. (Edinburgh: Oliver & Boyd, 1928. Price 15s. net.)

The appearance of a second edition of this work at an interval of three years after the publication of the first edition is an indication that it has met the needs of those who, in the course of research work, have to analyse large masses of data by statistical methods. The success of the work is doubtless due in large measure to the fact that it deals with what the statistical worker wants—"methods" and excludes proofs which in general involve the use of mathematics and of logical reasoning far beyond the average equipment of such workers. Those who require to study the proofs of the methods outlined in the book may find them in original papers or other publications, a detailed list of which is given.

The methods outlined in the book are illustrated by a large number of examples which serve to make clear the manner in which the methods are used. A series of tables given in the course of the text are reprinted on folders at the end, and can be cut out

and suitably mounted for practical use.

The first edition has been revised and some additions made. The most important of these is a new chapter dealing with the principles of statistical estimation. The table of z has been

considerably enlarged.

The volume forms one of a series of Biological Monographs, and has been written primarily with a view to biological workers. The subjects dealt with include tests of goodness of fit, independence and homogeneity; tests of significance of means, difference of means; regression and correlation coefficients; interclass and intraclass correlations; and the analysis of variance. All those who are concerned with such subjects will find the volume to be of great assistance to them.

Handbuch der Experimentalphysik. Herausgegeben von W. Wien und F. Harms. Bd. xviii. Wellenoptik und Polarisation, bearbeitet von K. F. Bottlinger, R. Ladenburg, M. v. Laue, Hans Schulz. Photochemie, von E. Warburg. [Pp. xiv+674, with 271 figures.] (Leipzig: Akademische Verlagsgesellschaft, m.b.H. 1928. Preis: brosch. 61,80; geb. 63,50 M.)

The new volume of the comprehensive treatise on experimental physics, edited by Profs. Wien and Harms, is devoted mainly to undulatory optics and the phenomena of the polarization of light. It includes sections on:—(1) The determination of the velocity of light in bodies at rest, by Prof. Ladenburg; (2) The optics of moving bodies, by Prof. v. Laue; (3) The relativity red displacement of special lines and gravitational deflexion of light, by Prof. Bottlinger; (4) The reflexion and refraction of light at the boundary between two isotropic media, by Prof. v. Laue; (5) The

interference and diffraction of electromagnetic waves (excluding Röntgen rays), by Prof. v. Laue; (6) The polarization of light, including crystal optics, by Prof. Schulz. The volume is completed by a monograph on photochemistry by Prof. Warburg; the original intention had been to include this in one volume with electrochemistry, but the delay in completion of the section devoted to the latter has necessitated a rearrangement.

Special emphasis is necessarily laid throughout on experimental methods and results. The theoretical aspect is nevertheless not overlooked, and receives adequate treatment. The volume is fully up to date, e.g. an account is given of Michelson's recent redetermination of the velocity of light. In an encyclopædic work of this nature, limitations of space hardly enter into consideration, and, with the exception of the relativity displacements of spectral lines and deflexion of light, each subject is dealt with in a comprehensive manner. The sections dealing with the velocity of light and with optical phenomena in moving bodies summarize all that has been done on these subjects. The sections dealing with interference, diffraction, and polarization are excellent, and contain a vast amount of information. Full references to original authorities are given throughout.

The printing and general production are of good quality, but the price, as in the case of most recent German publications, is high.

Chemical Encyclopædia. By C. T. KINGZETT, F.I.C., F.C.S. Fourth Edition. [Pp. viii. +807.] (London: Baillière, Tindall, & Cox. 1928. Price 35s. net.)

KINGZETT'S Chemical Encyclopædia is well known as a valuable work of reference for professional chemists, chemical engineers. manufacturers, students, etc. The whole of the material in the previous edition has been revised for the new issue, and extensive additions have been made, so that the new edition contains 200 more pages than the last. A valuable new feature is the inclusion of numerous bibliographical references, indicating where further details on any particular subject may be found. The usefulness of the work has thereby been greatly increased.

The subject-matter, as summarized by the compiler, comprises "pure, physical, and applied chemistry; descriptions of the elements and their chief compounds (organic and inorganic), as also the methods of their preparation, characters, and uses; brief accounts of ores and other natural products, together with their utilization: and the more important chemical terms and theories. In particular, increased attention has been given to chemical industries and the applications of chemistry in trades, the arts, and manufactures generally." There is no other work giving such varied information in a single volume; the references are remarkably complete and accurate. The author is to be congratulated on this new edition.

Magnetic Properties of Matter. By Kotaro Honda, D.Sc. [Pp. ix+256, with 213 figures.] (Tokyo: Syokwabo & Co. 1928. N. p.)

The original Japanese edition of this work was published in 1917. The English translation (by the author himself) will make available to a wider circle of readers a volume which is particularly valuable for the account of experimental work on the subject of the magnetic properties of matter. In this field of research Japanese investigators have taken a prominent place, and the original

publications are not always easily accessible.

The volume has been enlarged and revised where necessary in translation in order to take account of progress in the science of magnetism since the original Japanese edition. The chapter dealing with theories of magnetism has been rewritten and the additions include a chapter on the magnetic moment of atoms. The volume therefore contains a well-balanced account of the present state of observation and theory. The publishers deserve to be congratulated upon the excellence of the printing and general appearance of the volume.

Collected Researches, National Physical Laboratory. Vol. XX, 1927.

[Pp. v+444.] (London: H.M. Stationery Office. 1928.

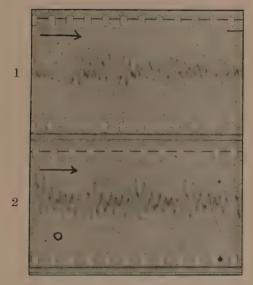
Price 18s. 6d. net.)

The latest volume of collected researches of the National Physical Laboratory contains reprints of papers by various members of the staff, the originals of which were published between 1920 and 1927 in various scientific publications. It is devoted mainly to geometrical optics, colorimetry, and photometry. Nearly one-half of the volume contains papers on various optical problems by Mr. T. Smith. Amongst the other contents may be noted papers dealing with the colour temperature and luminous efficiency relation for tungsten, with the performance and design of the optical system of ships' navigation lanterns and with methods of measurement by optical projection.

All the volumes of the 'Collected Researches,' with the exception of Volume I., which is now out of print, can be obtained from H.M. Stationery Office, Adastral House, Kingsway, W.C. 2. They form a valuable record of much of the important work carried on at the great Institution from which they emanate.

[The Editors do not hold themselves responsible for the views expressed by their correspondents.]

Frg. 11.



Vowel a: film 2 taken normally; film 1 taken with phases of components shifted.

There is no difference audible between 1 and 2.



Fig. 1.

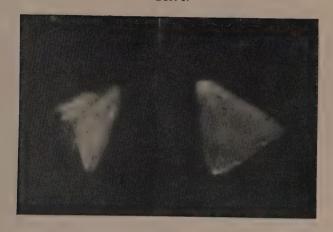
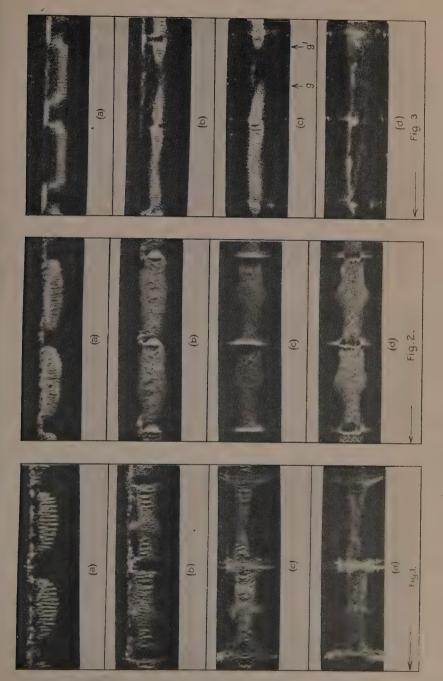


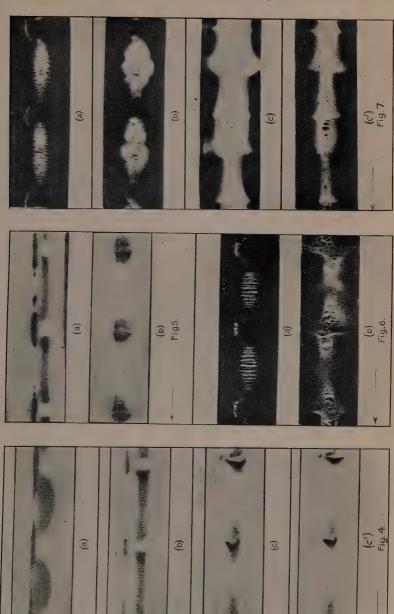
Fig. 2.



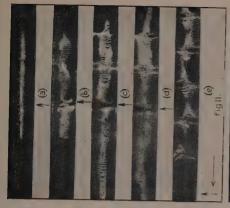


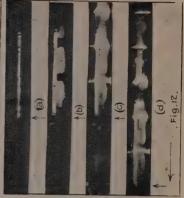


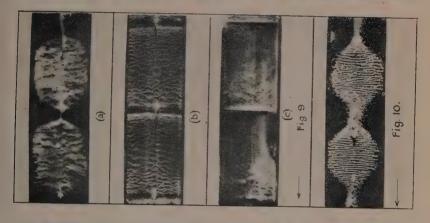


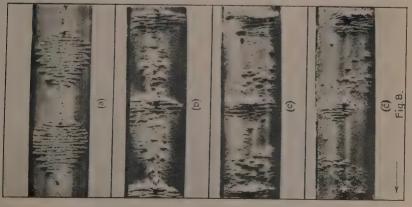




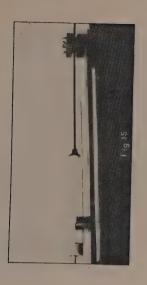


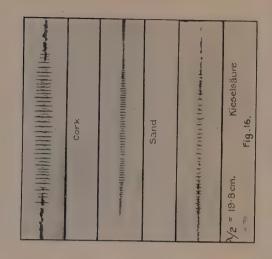




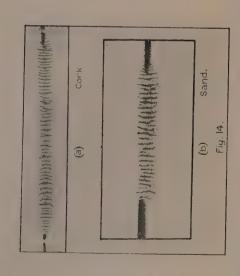




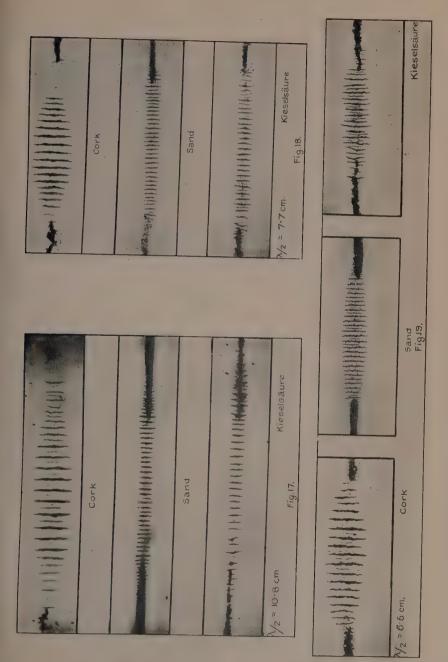








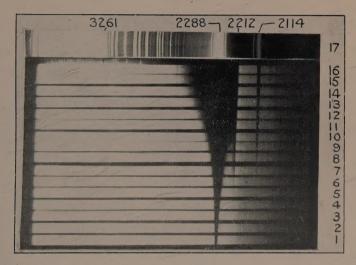






1:3

Fig. 1.



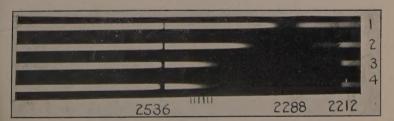
Band Spectrum of Cadmium. #1-16, Absorption Spectrum. #1, 3·3 mm.; #5, 14 mm.; #13, 95 mm.; #16, 170 mm. #17, Electrodeless Discharge in Cadmium.

Fig. 2.



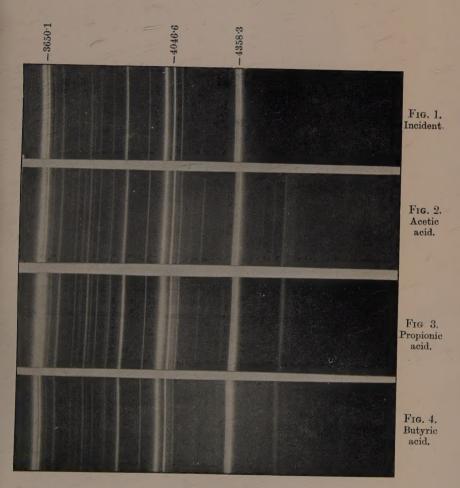
Absorption Spectrum of Zinc. #1,3 mm.; #2,5 mm.; #3,9 mm.

Fig. 3.



Absorption Spectrum of a Mercury-Cadmium Mixture. For #4, Hg at 100 mm., Cd at 60 mm.





Raman Effect in Fatty Acids.

